1. (15%) Solve $y'' - 4y' + 3y = -3\sin(2x+2)$; $y(-1) = 2$, $y'(-1) = 2$

2. (15%) Solve $x^2 y'' + 3xy' + y = 9x^2 + 8x + 5$

3. (15%) Solve the differential equation by Laplace transform.
   $ty'' + (4t - 2)y' - 4y = 0$; $y(0) = 1$, $y'(0) = 1$

4. (10%) Find all mathematical functions defined on $[-L, L]$ that are both even and odd where $L$ is an arbitrary constant.

5. (15%) Solve a one dimensional heat equation in a bar with length 10 and the heat coefficient $k$ is 2. If the temperature in left side of the bar is 20 degree and 30 degree in right side. What is the temperature distribution function $u(x,t)$ in the bar if the initial temperature profile is $q(x)$.

6. (15%) Solve a one dimensional heat equation in a bar with length $\pi$ and the heat coefficient $k$ is 10. If the both sides of the bar are insulated and the initial temperature in the bar is
   \[
   \begin{cases}
   u = 35 & \text{if } \pi/4 \leq x \leq 3\pi/4 \\
   u = 0 & \text{if } x < \pi/4 \text{ or } x > 3\pi/4
   \end{cases}
   \]

7. (15%) Solve the Dirichlet problem for a circular disk with radius $\pi$, assume the temperature around the disk is kept at 100 °C in upper half circle ($-\pi < \theta < \pi$).