大同大學 100 學年度研究所碩士班入學考試試題

考試科目：工程數學　　所別：電機工程研究所　　

註：本次考試 [不可以] 參考自己的書籍及筆記；[不可以] 使用字典；[不可以] 使用計算器。

Notation: $y' \equiv \dfrac{dy}{dt}$ and $y'' \equiv \dfrac{d^2y}{dt^2}$

1. Solve the differential equation: $(2xe^y + y\cos x) + (\sin x + x^2e^y - 1)y' = 0, \quad y(0) = 1.$ (15%)

2. Solve the initial value problem: $t\,y' + 2y = 4t^2, \quad y(1) = 2, \quad t > 0.$ (10%)

3. Find the general solution for the differential equation: $y'' - 6y' + 9y = 5e^{3x} + 2x + 1.$ (15%)

4. Solve the integral equation: $f(t) = e^{-3t} + \int_0^t f(t-\alpha)e^{-2\alpha}\,d\alpha.$ (10%)

5. Let $T : R^2 \to R^2$ be the linear operator defined by the equations

$$\begin{cases} v = x_1 + 2x_2 \\ w = -x_1 + x_2 \end{cases}$$

Determine whether T is one-to-one; if so, find $T^{-1}(v, w)$. (15%)

6. Let V be the space spanned by $v_1 = \cos^2 x$, $v_2 = \sin^2 x$, $v_3 = \cos 2x$. Determine whether $S = \{v_1, v_2, v_3\}$ is a basis for V. (10%)

7. Given the matrix $A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$, find A^{100}. (10%)

8. Given the linear system $Ax = b$

$$\begin{cases} x_1 - 2x_2 + x_3 + 2x_4 = -1 \\ 2x_1 - 4x_2 + 2x_3 + 6x_4 = 1 \\ -1x_1 + 2x_2 - x_3 - 2x_4 = 1 \end{cases}$$

(a) Determine a basis for the solution space of the system $Ax = 0$. (5%)

(b) Use the result in (a) to find the vector form of the general solution of the given system $Ax = b$. (10%)

THE END