註：本次考試 $\boxed{不可以}$ 參考自己的書籍及筆記； $\boxed{不可以}$ 使用字典； $\boxed{不可以}$ 使用計算器。

Notation: $y' \equiv \dfrac{dy}{dt}$ and $y'' \equiv \dfrac{d^2 y}{dt^2}$

1. Solve the following differential equation: $y' + \dfrac{2}{t} y = 4t^2$, $\quad t > 0$. (10%)

2. Find the solution of the differential equation: $y'' + y' - 2y = 2t$, $\quad y(0) = 0$, $\quad y'(0) = 1$. (15%)

3. Solve the differential equation: $y'' - 2y' - 3y = f(t)$, $\quad y(0) = 1$, $\quad y'(0) = 0$, with

$$f(t) = \begin{cases} 0 & \text{for } 0 \le t < 4 \\ 12 & \text{for } t \ge 4 \end{cases}.$$

(15%)

4. Solve the integral equation: $f(t) = t - 1 - 2\int_0^t f(\alpha)\sin(t - \alpha)\, d\alpha$. (10%)

5. Under what condition of θ, the matrix $\mathbf{A} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ has no real eigenvalue. (10%)

6. Let $\mathbf{A} = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$

    (a) find a matrix $\mathbf{P}$ that diagonalizes $\mathbf{A}$. (10%)
    (b) determine $\mathbf{P}^{-1}\mathbf{AP}$. (5%)

7. Solve the value of x_3 in the following equation by Cramer's rule,

$$\begin{cases} x_1 - 3x_2 + x_3 = 4 \\ 2x_1 - x_2 = -2 \\ 4x_1 - 3x_3 = 0 \end{cases}.$$

(15%)

8. Find a basis for the space spanned by the vectors (10%)

$\mathbf{v}_1 = (1, -2, 0, 0, 3)$, $\quad \mathbf{v}_2 = (2, -5, -3, -2, 6)$, $\quad \mathbf{v}_3 = (0, 5, 15, 10, 0)$, $\quad \mathbf{v}_4 = (2, 6, 18, 8, 6)$.

THE END