Notation: $y' \equiv \dfrac{dy}{dt}$ and $y'' \equiv \dfrac{d^2 y}{dt^2}$

1. Solve the following differential equation: $y' + 5t^4 y = t^4$, $y(0) = -7$. (10%)

2. Find the solution of the differential equation: $y'' - 3y' - 4y = 3e^{2t} + 2\sin t$. (15%)

3. Solve the differential equation: $y'' + 4y = h(t)$, $y(0) = 1$, $y'(0) = 0$, with

$$h(t) = \begin{cases} 0 & \text{for } t < 4 \\ 3 & \text{for } t \geq 4 \end{cases}.$$ (15%)

4. Solve the integral equation: $e^t \sin(y) - 2t + (e^t \cos(y) + 1)y' = 0$. (10%)

5. Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$. Assuming that $\det(A) = -2$, find (a) $\det(3A)$, (b) $\det(2A^{-1})$. (5%+5%)

6. Let $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$

   (a) Find the eigenvalues of matrix A. (5%)
   (b) Use the result in (a) to find the eigenspace corresponding to each eigenvalue. (10%)

7. Given the linear system $Ax = b$

$$\begin{cases} 3x_1 - 6x_2 + 3x_3 = -3 \\ 2x_1 - 4x_2 + 2x_3 = -2 \\ -1x_1 + 2x_2 - x_3 = 1 \end{cases}$$

   (a) Determine a basis for the solution space of the system $Ax = 0$. (5%)
   (b) Use the result in (a) to find the vector form of the general solution of the given stem $Ax = b$. (10%)

8. Let $T : R^2 \to R^2$ be the linear operator defined by the formula, $T(v, w) = (2v - w, v + w)$

   Determine whether T is one-to-one; if so, find $T^{-1}(v, w)$. (10%)

THE END