大同大學 103 學年度研究所碩士班入學考試試題

考試科目：工程數學　　所別：電機工程研究所　　

註：本次考試 [不可以] 參考自己的書籍及筆記；[不可以] 使用字典；[不可以] 使用計算器。

Notation: $y' \equiv \dfrac{dy}{dt}$ and $y'' \equiv \dfrac{d^2y}{dt^2}$

1. Solve the following differential equation: $y' + \dfrac{3}{t}y = 5t^2, \quad t > 0, \quad y(1) = 1.$ (10%)

2. Find the solution of the differential equation: $y'' - 4y' + 4y = e^{2x} + x + 1.$ (15%)

3. Solve the differential equation: $y'' + 5y' + 6y = g(t);\ y(0) = y'(0) = 0,$ with

$$g(t) = \begin{cases} -2 & \text{for } 0 \le t < 3 \\ 0 & \text{for } t \ge 3 \end{cases}.$$ (15%)

4. Solve the integral equation: $f(t) = 2 + \int_0^t f(t-\alpha)\cos(4\alpha)\, d\alpha.$ (10%)

5. Let **A, B** and **C** be any 2×2 real matrices. Label the following statements as being true or false and explain it detail.

(a) If **AB** = **AC**, then **B** = **C**. (5%)

(b) If **AB = 0**, then **A = 0** or **B = 0.** (5%)

6. Suppose that $T : R^2 \to R^2$ is linear operator and that $T(1,0) = (1,4)$ and $T(1,1) = (2,5)$.

(a) What is $T(2,3)$? (5%)

(b) Is T one-to-one. (5%)

7. Let $A = \begin{bmatrix} 0 & -2 & -3 \\ -1 & 1 & -1 \\ 2 & 2 & 5 \end{bmatrix}$.

(a) Find a matrix P that diagonalizes the matrix A. (10%)

(b) Determine $P^{-1}AP$. (5%)

8. Solve the following system $Ax = b$ by Cramer's rule. (15%)

$$\begin{cases} 3x_1 + x_2 + x_3 = 4 \\ -2x_1 - x_2 = 12 \\ x_1 + 2x_2 + x_3 = -8 \end{cases}$$

THE END