1. Solve the following initial value problem

$$\frac{dy(t)}{dt} = \sin(t) + \int_0^t y(\tau)\cos(t-\tau)d\tau, \qquad y(0) = 2.$$

2. The LC circuit is given. Before time $t = 0$ the switch S is open, and zero initial current and zero charge are assumed. Next, the switch S is closed at time $t = 0$.

   (a) Write the differential equation for the current $i(t)$.

   (b) When $L = 10$ henry, $C = 0.025$ farad, and the input $v(t) = 10\cos(3t)u(t)$ volts. (where $u(t)$ is the unit step function), find the current $i(t)$.

   (c) $L = 1$ henry, $C = 1$ farad, the input $v(t) = 1$ volt when $0 < t < 1$, and the input $v(t) = 0$ volt when $t > 1$, find the current $i(t)$.
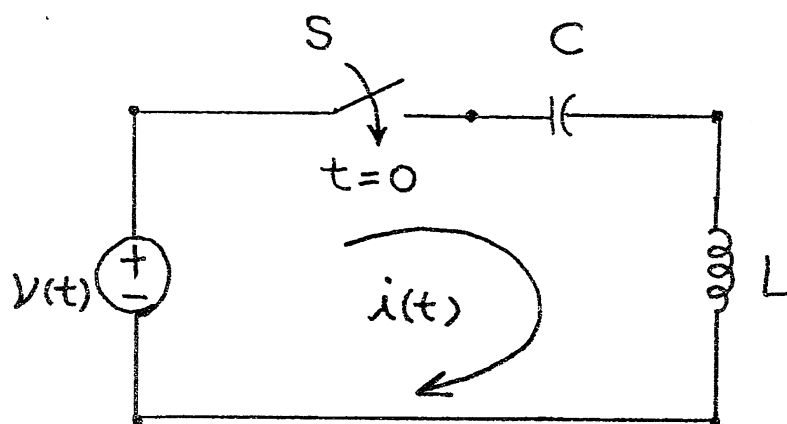
3. Let the Fourier-like transform of $f(t)$ be defined by

$$F(\omega) = \Im\{f(t)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)\,e^{j\omega t}dt, \qquad \text{where } j = \sqrt{-1}.$$

Show the detailed derivation to find the Fourier-like transform of the following functions

   (1) $\Im\{\delta(t-3)\} = ?$, where $\delta(t)$ is unit impulse function.

   (2) $\Im\{e^{j2t}\} = ?$

   (3) $\Im\{\cos(\omega_0 t)\} = ?$

$\langle$ TO　BE　CONTINUED $\rangle$

4. Let $T : R^2 \to R^2$ be the linear transformation defined by the formula

$$T(x, y) = (x - y, 2x + 4y)$$

- (a) Determine whether T is one-to-one; if so, find $T^{-1}(x, y)$.
- (b) Find the eigenvalues and the corresponding eigenvectors of T.

5. Let R^2 have inner product $< (u_1, u_2), (v_1, v_2) > = 3 u_1 v_1 + u_2 v_2$.

- (a) Verify that the vectors $u = (\frac{1}{2}, \frac{1}{2})$ and $v = (\frac{1}{2\sqrt{3}}, \frac{-3}{2\sqrt{3}})$ form an orthonormal basis for R^2.

- (b) Express the vector $w = (3, 7)$ as a linear combination of u and v.

6. The joint probability density function, $f(x, y)$, for a random variable X and a random variable Y is given by:

$$f(2,1) = 0.10, \quad f(2,3) = 0.20, \quad f(2,5) = 0.10,$$
$$f(4,1) = 0.15, \quad f(4,3) = 0.30, \quad f(4,5) = 0.15,$$

(a) Find the expected value of $g(X, Y) = XY$.

(b) Find the covariance of X and Y.

題號	1	2	3	4	5	6
配分	16	18	18	16	16	16