# 大同大學九十一學年度研究所碩士班入學考試試題

考試科目：控制系統　　所別：電機工程研究所　　

註：本次考試 不可以 參考自己的書籍及筆記； 不可以 使用字典； 不可以 使用計算器。

1. A linear time-invariant system is described by the differential equation

$$\frac{d^3y(t)}{dt^3}+3\frac{d^2y(t)}{dt^2}+3\frac{dy(t)}{dt}+y(t)=r(t)$$

(a) Let the state variables be defined as $x_1(t)=y(t)$ , $x_2(t)=y(t)+\frac{dy(t)}{dt}$ , and $x_3(t)=y(t)+2\frac{dy(t)}{dt}+\frac{d^2y(t)}{dt^2}$. Write the state equations of the system in vector-matrix form: $\dot{\mathbf{x}}=\mathbf{A}\mathbf{x}+\mathbf{B}r$, $y=\mathbf{C}\mathbf{x}$, where $\dot{\mathbf{x}}\equiv[\,\dot{x}_1\ \ \dot{x}_2\ \ \dot{x}_3\,]^{\mathrm{T}}\equiv[\,dx_1/dt\ \ dx_2/dt\ \ dx_3/dt\,]^{\mathrm{T}}$. (6%)

(b) Find the state-transition matrix $\phi(t)$ of the state equation obtained in (a). (6%)

(c) Find the impulse response of the system. (4%)

2. The OP-amp circuit implementation of a phase-lead controller is given in Fig. 1, where $C=0.1\,\mu F$, $R_1=500\,k\Omega$, $R_2=40\,k\Omega$.

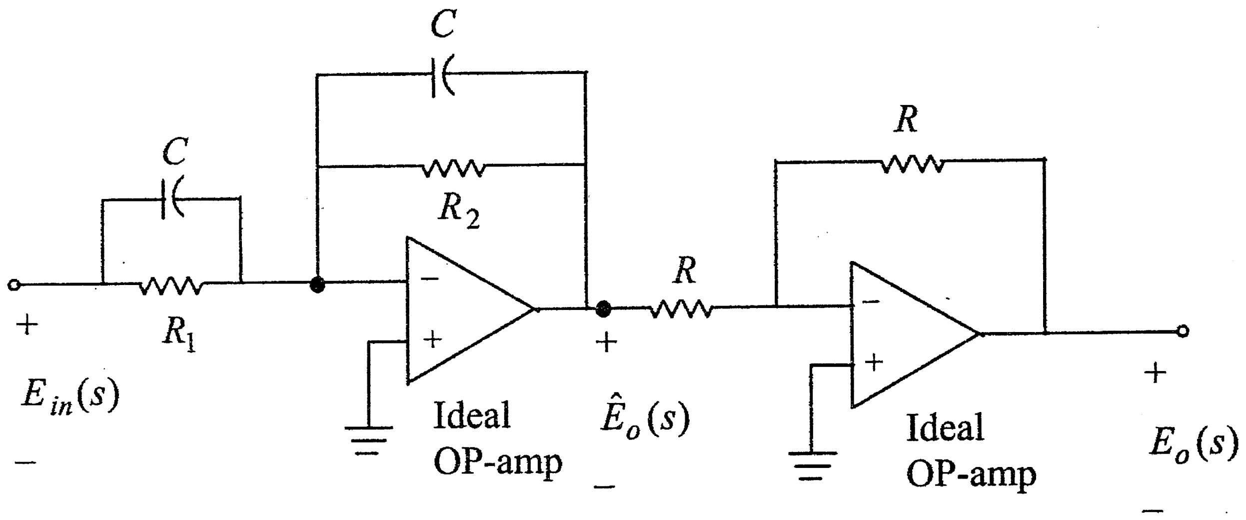


Fig. 1

(a) Find the gain of the controller $\hat{G}_c(s)=\frac{\hat{E}_o(s)}{E_{in}(s)}$ and $G_c(s)=\frac{E_o(s)}{E_{in}(s)}$. (12%)

(b) Find the zero and the pole of $G_c(s)$. (4%)

3. Given the forward-path transfer function of a negative unity-feedback control system

$$G(s)=\frac{K(s+3.15)}{s(s+1.5)(s+0.5)}$$

(a) Determine the type of the system. (2%)

(b) Determine the step, ramp, and parabolic error constants of the system. (6%)

(c) Determine the steady-state error for a unit-step input, a unit-ramp input, and a parabolic input, $(t^2/2)u_s(t)$. (Note: $u_s(t)$ denotes the unit-step function) (6%)

TO BE CONTINUED ↷

4. A controlled process is modeled by the following state equations.

$$\frac{dx_1(t)}{dt} = x_1(t) - 2x_2(t), \quad \frac{dx_2(t)}{dt} = 10x_1(t) + r(t)$$

(a) Determine the stability of the process. (6%)

(b) If the control input $r(t)$ is obtained from state feedback, such that

$$r(t) = -k_1 x_1(t) - k_2 x_2(t)$$

where $k_1$ and $k_2$ are real constants. Determine the region in the $k_1$ versus $k_2$ parameter plane in which the closed-loop system is asymptotically stable. (Use $k_2$ as the vertical axis and $k_1$ as the horizontal axis.) (12%)

5. Consider the network system shown in Fig. 2.

(a) Draw a signal flow graph based on the input node $v_s$, the variables node $I_1$, $v_2$, $I_3$ and the output node $v_o$. (6%)

(b) Find $\frac{v_o}{v_s}$ by Mason's gain formula. (5%)

(c) Find $\frac{I_3}{I_1}$ by Mason's gain formula. (5%)

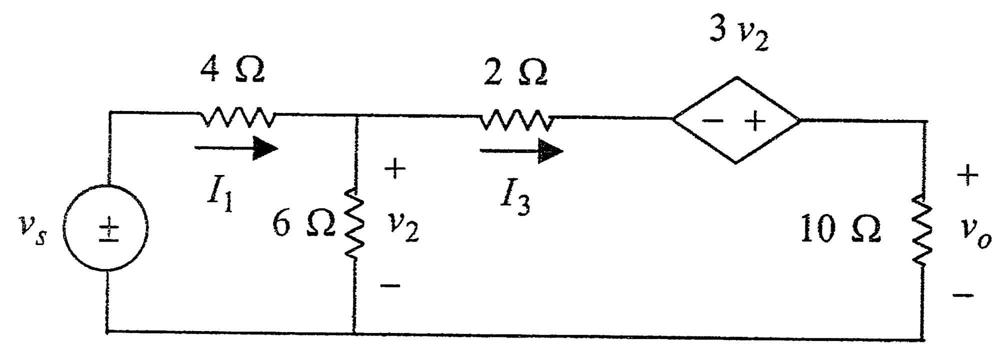


Fig. 2

6. Consider the sampled-data system shown in Fig 3. If the input

$$r(t) = \begin{cases} 1 - \cos(t) & 0 \le t < 1.2 \text{ sec} \\ 0 & \text{otherwise} \end{cases}$$

(a) Find $R(z) = \mathcal{Z}[r^*(t)]$. (Note: $\mathcal{Z}[r^*(t)]$ denotes the Z-transform of $r^*(t)$ ) (5%)

(b) Draw the waveform of $\bar{y}(t)$. (5%)

(c) Find the value of $y(\pi/2)$. (4%)

(d) Find the value of $y(t)$ as $t \to \infty$. (4%)

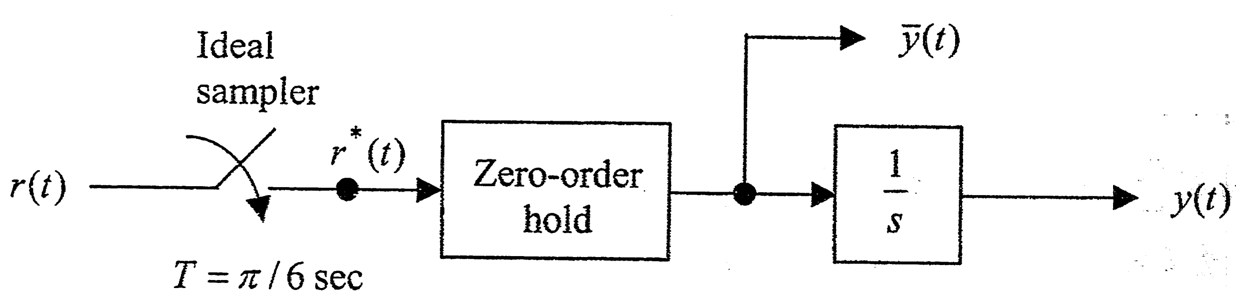


Fig. 3

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