大同大學 九十三 學年度研究所碩士班入學考試試題

考試科目：工程數學　　所別：電機工程研究所　　第 1/2 頁

註：本次考試 不可以參考自己的書籍及筆記； 不可以使用字典； 不可以使用計算器。

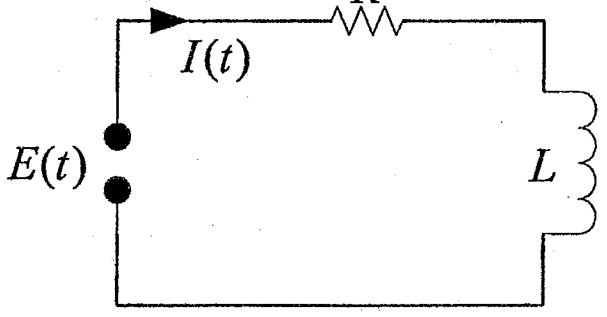
1. Solve the following system of differential equations. (16%)

$$\begin{cases} \dfrac{dy_1}{dt} = 6y_1 + 9y_2 \quad , \quad y_1(0) = -3 \\ \dfrac{dy_2}{dt} = y_1 + 6y_2 \quad , \quad y_2(0) = -3 \end{cases}$$

2. The RL circuit is given.

(a) Write the differential equation for the current $I(t)$. (6%)

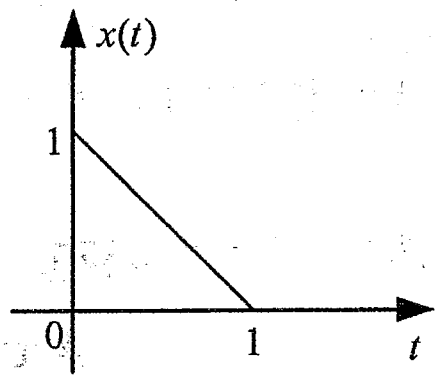
(b) Let $R = 1$ ohm, $L = 1$ henry, and the input $E(t) = 1$ volt, when $0 < t < 3$ sec, and $E(t) = 0$, when $t > 3$ sec. Find the current $I(t)$, assuming $I(0) = 0.5$ ampere. (12%)



3. (a) For the function $x(t)$, find its Fourier transform $X(\omega)$. (6%)

(b) Find the value of $\int_{-\infty}^{\infty} |X(\omega)|^2 \, d\omega$. (6%)

(c) Find the Fourier transform for the function $X(2-t)$. (6%)



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4. Consider the basis $S=\{\vec{v}_1,\vec{v}_2,\vec{v}_3\}$ for R^3, where

$$\vec{v}_1=(1,1,1),\ \vec{v}_2=(1,1,0),\ \text{and}\ \vec{v}_3=(1,0,0),$$

and let $T:R^3\to R^3$ be the linear operator such that

$$T(\vec{v}_1)=(2,-1,4),\ T(\vec{v}_2)=(3,0,1),\ T(\vec{v}_3)=(-1,5,1).$$

(a) Find a formula for $T(x_1,x_2,x_3)$. (10%)

(b) Determine $T(2,4,-1)$. (6%)

5. (a) Find a subset of the vectors

$$\vec{v}_1=(1,-1,5,2),\ \vec{v}_2=(-2,3,1,0),\ \vec{v}_3=(4,-5,9,4)$$

$$\vec{v}_4=(0,4,2,-3),\ \vec{v}_5=(-7,18,2,-8)$$

that forms a basis for the space spanned by the vectors. (8%)

(b) Express each vector that is not in the basis as a linear combination of the basis vectors. (8%)

6. $f(x,y)$ is a joint probability density function,

$$E(X)=3,E(Y)=7,E(X^2)=13,E(Y^2)=50,\ \text{and}\ E(XY)=22.$$

(a) Are X and Y independent ? Why or why not ? (3%)

(b) What are the mean and variance of $W=12X+3$? (4%)

7. The probabilities for X and Y can be found from the function:

$$f(x,y)=kx,\quad 0<x<y<2.$$

(a) Find the value of k. (3%)

(b) Find the marginal probability of X. (3%)

(c) Find the probability $P(X<1/Y=1.5)$. (3%)