大同大學 九十四 學年度研究所碩士班入學考試試題

考試科目:工程數學　　所別:電機工程研究所　　

註:本次考試不可以參考自己的書籍及筆記；不可以使用字典；不可以使用計算器。

1. Let V be an inner product space and the set of vectors $\{v_1, v_2, \dots, v_N\} \subseteq V$. 【15 %】

(a) Prove that $v_1, v_2, \dots, v_N$ are linearly independent if $\{v_1, v_2, \dots, v_N\}$ is an orthogonal set.
(b) Find an orthogonal basis for the space spanned by $\{(1,1,1,1), (1,1,0,0), (0,1,1,0)\}$ in $\mathbb{R}^4$.

2. Find all values of a for which the following system is consistent. 【10 %】

$$x + y - z = 1$$
$$x + 2y + z = 3$$
$$x + y + (a^2 - 2)z = a$$

3. Find the general solution of the following differential equation. 【10 %】

$$\frac{dy}{dx} = \frac{y}{x} + \frac{x}{y}$$

4. Find the general solution of the following system. 【15 %】

$$\frac{dx_1}{dt} = -2x_1 + x_2$$
$$\frac{dx_2}{dt} = -4x_1 + 3x_2 + 10\cos(t)$$

5. Consider a **real** function $x(t)$ whose Fourier transform is $X(\omega)$. Prove that the real part of $X(\omega)$ is an even function and the imaginary part of $X(\omega)$ is an odd function. 【15 %】

6. Find the the Fourier transform of the function $e^{-\beta|t|}$, where $\beta > 0$ is a constant. 【10 %】

7. Suppose that the random variables X and Y have the joint probability density function, $P(X = x, Y = y) \equiv f(x, y)$, defined as follows:

$$f(x,y) = \begin{cases} \dfrac{ky}{x^3}, & 2 < x\ ;\ 0 < y < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find the marginal distributions for X and Y. 【10 %】
(b) Evaluate the correlation coefficient ρ_{XY}. 【5 %】

8. Let X_1 and X_2 be two random variables with the joint probability function, $P(X_1 = x_1, X_2 = x_2) \equiv f(x_1, x_2)$, defined as follows:

$$f(x_1, x_2) = \begin{cases} \dfrac{x_1 x_2}{k}, & x_1 = 1,\ 2;\ x_2 = 1,\ 2,\ 3 \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find the value of k. 【5 %】
(b) Find the probability function of the random variable $Y = X_1 X_2$. 【5 %】