大同大學 95 學年度研究所碩士班入學考試試題

考試科目：工程數學　　　**所別：電機工程研究所**　　　

註：本次考試 不可以 參考自己的書籍及筆記； 不可以 使用字典； 不可以 使用計算器。

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1. Solve the initial value problem: $y' + \dfrac{2}{t+1} y = 3$; $y(0) = 5$. (**Note:** $y' \equiv \dfrac{dy}{dt}$) (8%)

2. Find the general solution of the following differential equation

$y'' + 4y' + 4y = 2\cos(2t) - 3te^{-2t}$ (**Note:** $y' \equiv \dfrac{dy}{dt}$ and $y'' \equiv \dfrac{d^2y}{dt^2}$) (10%)

3. Solve the integral equation: $f(t) = e^{-3t}\left[e^t - 3\int_0^t f(a)e^{3a}\,da\right]$ (8%)

4. Let R^4 have the Euclidean inner product. Find two vectors of norm 1 that are orthogonal to the three vectors $u = (2, 1, -4, 0)$, $v = (-1, -1, 2, 2)$, and $w = (3, 2, -3, 4)$. (12%)

5. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

(1) Find the roots of the characteristic equation of A. (6%)

(2) Find the sufficient condition such that the matrix A is diagonalizable and explain it. (8%)

6. For the continuous-time periodic signal $x(t)$

$$x(t) = \begin{cases} 2, & 0 \le t \le 2 \\ -1, & 2 \le t < 4 \end{cases} \quad \text{and} \quad x(t+4) = x(t)$$

determine the fundamental frequency w_0 and the Fourier series coefficients a_k such that

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0 t}.$$ (12%)

7. Let $X(w)$ denotes the Fourier transform of $x(t)$.

(1) Find the Fourier transforms of $\dfrac{d}{dt}x(t-3)$ in terms of $X(w)$ (6%)

(2) Find the inverse Fourier transforms of $X^*(w_0 - w)$ in terms of $x(t)$. (6%)

(w_0 is a constant)

TO BE CONTINUED

8. A random variable *X* has a probability density function

$$f(x) = \begin{cases} 1/(b-a) & a \le x \le b \\ 0 & \text{eleswhere} \end{cases}, \text{ where } a \ne b$$

(a) Find the distribution function of the random variable *X*. (3%)

(b) Find the mean value of the random variable *X*. (3%)

(c) Find the variance of the random variable *X*. (3%)

(d) Find the variance of the new random variable $Y = 4X + 2$. (3%)

9. Two random variables X and Y have means $E[X] = 1$ and $E[Y] = 2$, variances $s_X^2 = 4$ and $s_Y^2 = 1$, and a correlation coefficient $r_{XY} = 0.4$. New random variables *V* and *W* are defined by

$$V = -X + 2Y \qquad W = X + 3Y$$

(a) Find the variance of *V*. (6%)

(b) Find the $E[VW]$. (6%)

Note: $E[Z]$ denotes the mean value of the random variable *Z*.

THE END

1. Solve the initial value problem $y' + \frac{2}{x+1} y = 3\,;\quad y(0) = 5.$ P.27 (8%)

$Ans: y = x + 1 + 4(x+1)^{-2}$

2. Find the general solutions of the following differential equation p.97

$$y'' + 4y' + 4y = 2\cos(2x) - 3xe^{-2x} \qquad (10\%)$$

$$Ans: y = c_1 e^{-2x} + c_2 x e^{-2x} + \frac{1}{4}\cos(2x) - \frac{1}{2}x^3 e^{-2x}$$

3. Solve the integral equation $f(t) = e^{-3t}\left[e^t - 3\int_0^t f(a)\, e^{3a}\, da\right]$ p.146 (8%)

$$Ans: F(s) = \frac{s+3}{(s+2)(s+6)} = \frac{1}{4(s+2)} + \frac{3}{4(s+6)}$$

$$\Rightarrow f(t) = \frac{1}{4}e^{-2t} + \frac{3}{4}e^{-6t}$$

3. Solve the integral equation $f(t) = e^{-3t}\left[e^{t} - 3\int_0^t f(a)\,e^{3a}\,da\right]$ p.146 (8%)

$$Ans: F(s) = \frac{1}{s+2} - \frac{3}{s+3}F(S)$$

$$\Rightarrow \frac{s+6}{s+3}F(S) = \frac{1}{s+2} \Rightarrow F(s) = \frac{s+3}{(s+2)(s+6)} = \frac{1}{4(s+2)} + \frac{3}{4(s+6)}$$

$$\Rightarrow f(t) = \frac{1}{4}e^{-2t} + \frac{3}{4}e^{-6t}$$

8.