大同大學 96 學年度研究所碩士班入學考試試題

註：本次考試 不可以 參考自己的書籍及筆記； 不可以 使用字典； 不可以 使用計算器。

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1. Solve the general solution of the following differential equation:

$$y' - \frac{2}{t} y = t^2 \sin(4t). \quad (\textbf{Note:}\ y' \equiv \frac{dy}{dt}) \qquad (8\%)$$

2. The general solution of the nonhomogeneous differential equation $y'' + \alpha y' + \beta y = g(t)$ is given as $y(t) = c_1 e^t + c_2 t e^t + t^2 e^t$, where c_1 and c_2 are arbitrary constants. Determine the constant α and β and the function $g(t)$. (**Note:** $y' \equiv \frac{dy}{dt}$ and $y'' \equiv \frac{d^2 y}{dt^2}$) (10%)

3. Find the inverse Laplace transform for the given $F(s)$.

$$F(s) = \frac{e^{-5s}}{s^2+1} + \frac{s+4}{s^2+4} + \frac{1}{s+2}. \qquad (8\%)$$

4. Let $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$.

(a) Find all eigenvalues of A. (5%)

(b) Find a maximum set S of linearly independent eigenvectors of A. (5%)

(c) Is A diagonalizable ?

(**Note:** If yes, find P such that $D = P^{-1} AP$ is diagonal; if no, state the reasons.) (5%)

5. Find the matrix representation of each of the following linear operators F on R^3 relative to the usual basis $E = \{e_1, e_2, e_3\}$ of R^3; that is, find $[F] = [F]_E$:

(a) F defined by $F(x, y, z) = (x + 2y - 3z,\ 4x - 5y - 6z,\ 7x + 8y + 9z)$. (5%)

(b) F defined by the 3×3 matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 5 & 5 & 6 \end{bmatrix}$. (5%)

(c) F defined by $F(e_1) = (1, 3, 5), F(e_2) = (2, 4, 6), F(e_3) = (7, 7, 8)$. (5%)

TO BE CONTINUED

6. Let $x(t)=\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$ be the Fourier series representation of the continuous-time periodic signal $x(t)=\begin{cases}1, & 0\le t<1\\ 3, & 1\le t\le 2\\ 0, & 2\le t<4\end{cases}$ and $x(t+4)=x(t)$.

(a) Find the fundamental frequency ω_0. (6%)

(b) Find the Fourier coefficients a_0 and a_2. (6%)

7. Let $X(j\omega)$ be the Fourier transform of the continuous-time signal $x(t)$.

$$X(j\omega)=\begin{cases}\pi, & |\omega|\le 1\\ 0, & \text{otherwise}\end{cases}.$$

(a) Find $x(t)$. (6%)

(b) $\int_{-\infty}^{\infty} |x(t)|^2 dt = ?$ (6%)

8. Let W be a random variable equal to the sum of two independent random variables X and Y:

$$W=X+Y,$$

where the density functions of X and Y are assumed to be

$$f_X(x)=\begin{cases}1/4 & 0\le x\le 4\\ 0 & \text{eleswhere}\end{cases}, \quad f_Y(y)=\begin{cases}1/2 & 0\le y\le 2\\ 0 & \text{eleswhere}\end{cases}.$$

(a) Find the density function of the random variable W. (4%)

(b) Find the mean value of the random variable W. (4%)

9. A random variables X has mean $E[X]=-3$ and variance $\sigma_X^2=2$. A new random variable Y is defined by

$$Y=2X-3.$$

(a) Find $E[X^2]$. (4%)

(b) Find $E[Y^2]$. (4%)

(c) Find σ_Y^2. (4%)

Note: $E[Z]$ denotes the mean value of the random variable Z.

THE END