

大同大學 97 學年度研究所碩士班入學考試試題

考試科目：工程數學

所別：電機工程研究所

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註：本次考試 不可以 參考自己的書籍及筆記； 不可以 使用字典； 不可以 使用計算器。

1. Solve the initial value problem: $y' + 2ty = 4t$; $y(0) = 3$. (Note: $y' \equiv \frac{dy}{dt}$) (10%)

2. Find the general solution of the following differential equation

$$4y'' - 4y' + y = 0, \quad y(1) = -4, \quad y'(1) = 0. \quad (\text{Note: } y' \equiv \frac{dy}{dt} \text{ and } y'' \equiv \frac{d^2y}{dt^2}) \quad (10\%)$$

3. Find the convolution of the following two signals:

$$f(t) = \begin{cases} e^{-3t}, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}, \quad g(t) = \begin{cases} 1, & 1 \leq t \leq 5 \\ 0, & \text{otherwise} \end{cases} \quad (10\%)$$

4. Find the general solution of the nonhomogeneous linear system.

$$\begin{cases} x_1 + 3x_2 - 2x_3 + 2x_5 = 0 \\ 2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 = -1 \\ 5x_3 + 10x_4 + 15x_6 = 5 \\ 2x_1 + 6x_2 + 8x_4 + 4x_5 + 18x_6 = 6 \end{cases} \quad (12\%)$$

5. Let $T: R^3 \rightarrow R^3$ be the linear operator defined by the formula,

$$T(x_1, x_2, x_3) = (3x_1 + x_2, -2x_1 - 4x_2 + 3x_3, 5x_1 + 4x_2 - 2x_3).$$

Determine whether T is one-to-one, if so, find $T^{-1}(x_1, x_2, x_3)$. (15%)

6. Consider the continuous-time periodic signal $x(t) = 1 - 2\sin(3t) + 3\cos(2t)$.

(a) Find the fundamental frequency ω_0 of the waveform $x(t)$. (5%)

(b) Find the Fourier series coefficients a_k such that $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$. (5%)

(c) Find the average power of $x(t)$. (5%)

TO BE CONTINUED 

7. Let $X(\omega)$ be the Fourier transform of the continuous-time waveform $x(t) = e^{-2t}U(t)$, where $U(t)$ is the unit step function.

(a) Find $X(2)$, i.e., the Fourier transform at the frequency $\omega = 2$. (5%)

(b) Find the magnitude and phase angle of $X(2)$. (5%)

8. A random variable X has a probability density function

$$f(x) = \begin{cases} 1/10, & 0 \leq x \leq 10 \\ 0, & \text{elsewhere} \end{cases}$$

(a) Find the mean value of the random variable X . (3%)

(b) Find the variance of the random variable X . (3%)

(c) Find the variance of the new random variable $Y = 3X + 1$. (3%)

9. You (person A) and two others (B and C) each toss a fair coin in two-step gambling game. In step 1 the person whose toss is not a match to either of the other two is "odd man out." Only the remaining two whose coins match go on to step 2 to resolve the ultimate winner.

(a) What is the probability you will advance to step 2 after the first toss? (3%)

(b) What is the probability you will be out after the first toss? (3%)

(c) What is the probability that no one will be out after the first toss? (3%)

THE END