1. Solve the initial value problem: $ty' + 4y = 6t^2$, $y(1) = 2$, $t > 0$. (**Note:** $y' \equiv \dfrac{dy}{dt}$) (15%)

2. Find the general solution of the following differential equation

$$y'' + 2y' + y = 2e^{-t}.$$ (**Note:** $y' \equiv \dfrac{dy}{dt}$ and $y'' \equiv \dfrac{d^2 y}{dt^2}$) (15%)

3. Let

$$A = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 4 & 5 \\ 0 & 0 & 9 \end{bmatrix}.$$

(a) Find a matrix P that diagonalizes the matrix A. (10%)

(b) Find a matrix S such that $S^2 = A$. (8%)

4. What conditions must be satisfied by $b_1, b_2, b_3, b_4,$ and b_5 for the following linear system to be consistent? (10%)

$$\begin{cases} x_1 - 2x_2 = b_1 \\ x_1 - x_2 = b_2 \\ x_1 + x_2 = b_3 \\ x_1 + 2x_2 = b_4 \\ x_1 + 3x_2 = b_5 \end{cases}.$$

TO BE CONTINUED

5. Consider the continuous-time periodic signal

$$x(t) = 1 - 2\sin(6t) + 4\cos(4t) + 6\cos(2t).$$

And let a_k denote its Furier series coefficients such that $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$.

(a) Find the fundamental frequency and the fundamental period for $x(t)$. (6%)

(b) Find the Furier series coefficients a_k. (6%)

(c) Find the average power of $x(t)$. (6%)

6. Let $X(j\omega)$ denote the continuous-time Fourier transform (CTFT) of the aperiodic signal

$$x(t) = e^{-2t}u(t-3),$$

where $u(t)$ is the unit step function. And let $\hat{x}(t)$ denote the inverse Fourier transform of $X(j\omega)$.

(a) Find $X(j\omega)$. (6%)

(b) Find $\hat{x}(1)$, $\hat{x}(3)$, and $\hat{x}(5)$. (6%)

7. A random variable X has a probability density function

$$f(x) = \begin{cases} a, & 0 \le x \le 20 \\ 0, & \text{eleswhere} \end{cases}.$$

(a) Find the value of a. (3%)

(b) Find the mean value of the random variable X. (3%)

(c) Find the variance of the random variable X. (3%)

(d) Find the variance of the new random variable $Y = 2X + 3$. (3%)

THE END