註：本次考試 不可以 參考自己的書籍及筆記； 不可以 使用字典； 不可以 使用計算器。

1. If the nonhomogeneous differential equation $y'' + \alpha y' + \beta y = r(t)$ has the general solution $y(t) = c_1 e^{-2t} + c_2 t e^{-2t} + 3t^2 e^{-2t}$, where c_1 and c_2 are arbitrary constants, determine the constant α and β and the function $r(t)$. (**Note:** $y' \equiv \dfrac{dy}{dt}$ and $y'' \equiv \dfrac{d^2 y}{dt^2}$) (10%)

2. Find the general solution for the differential equation: $t^2 y'' + 2ty' - 12y = 0$. (10%)

   (**Note:** $y' \equiv \dfrac{dy}{dt}$ and $y'' \equiv \dfrac{d^2 y}{dt^2}$)

3. Solve the initial value problem: $y' + 2ty = 6t$, $y(0) = 7$. (**Note:** $y' \equiv \dfrac{dy}{dt}$) (10%)

4. Solve the integral equation: $f(t) = 3 + \displaystyle\int_0^t f(\alpha) \cos[2(t - \alpha)] \, d\alpha$ (7%)

5. Use the Laplace transform to solve the following system

   $$\begin{cases} x'' - 2x' + 3y' + 2y = 4, \\ 2y' - x' + 3y = 0, \end{cases} \quad x(0) = x'(0) = y(0) = 0.$$

   (**Note:** $x' \equiv \dfrac{dx}{dt}, x'' \equiv \dfrac{d^2 x}{dt^2}$ and $y' \equiv \dfrac{dy}{dt}$) (10%)

6. Let $x(t) = \begin{cases} e^{-2t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$ and $h(t) = \begin{cases} e^{-t} + e^{-2t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$.

   Find the convolution $x(t) * h(t)$ of $x(t)$ with $h(t)$. (8%)

TO BE CONTINUED ↷

7. Give the data

i	1	2	3	4	5
(x_i, y_i)	$(1,2)$	$(2,3)$	$(3,5)$	$(4,9)$	$(5,1)$

Find the least squares straight line (in the form of $y = a + bx$) fit to these five points.　　　(10%)

8. Consider the polynomials $f(t) = t + 2$ and $g(t) = 3t - 2$ in $\mathbf{P}(t)$ (the set of all polynomials with real coefficients) with the inner product

$$< f, g > = \int_0^1 f(t)g(t)dt$$

(a) Find $< f, g >$.　　　(5%)

(b) Find $\| f \|$ and $\| g \|$ (the norm of f and g).　　　(10%)

9. Let $T : R^3 \to R^3$ be defined by $T(x, y, z) = (2x + y - 2z,\ 2x + 3y - 4z,\ x + y - z)$

(a) Find all eigenvalues of T.　　　(8%)

(b) Find a basis of each eigenspace corresponding to (a).　　　(8%)

(c) Is T diagonalizable?　(State the reasons).　　　(4%)

THE   END