Part I (50%)

1. Determine the equao n of the polynomial of degree three whose graph passes through the points (1, -3), (2, -1), (3, 9), (4, 33). You must

(a) set up the system of linear equao ns, (7%)

(b) solve the system by Gauss-Jordan eliminao n. (3%)

2. If $A = \begin{bmatrix} 5 & 1 \\ 9 & 2 \end{bmatrix}$ then $A^{-1} = \begin{bmatrix} 2 & -1 \\ -9 & 5 \end{bmatrix}$. Use this informao n to determine (6%,每題 3%)

(a) $(2A^t)^{-1}$ (b) $(AA^t)^{-1}$

3. If A and B are 3× 3 matrices and $|A| = -3$, $|B| = 2$, compute the following determinants. (18%,每題 3%)

(a) $|AB|$ (b) $|AA^t|$ (c) $|A^tB|$

(d) $|3A^2B|$ (e) $|2AB^{-1}|$ (f) $|(A^2B^{-1})^t|$

4. If $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $|A| = 3$, compute the determinants of the following matrices. (8%,每題 2%)

(a) $\begin{bmatrix} d & e & f \\ g & h & i \\ a & b & c \end{bmatrix}$ (b) $\begin{bmatrix} d & a & g \\ e & b & h \\ f & c & i \end{bmatrix}$ (c) $\begin{bmatrix} d & f & e \\ a & c & b \\ g & i & h \end{bmatrix}$ (d) $\begin{bmatrix} d & e & f \\ a & b & c \\ 2g & 2h & 2i \end{bmatrix}$

5. Find the reduced echelon form for each of the following matrices. Use the echelon form to determine a basis for the row space, and the rank of each matrix. (8%,每題 4%)

(a) $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 5 & 2 \\ 0 & 2 & 9 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 & 8 \\ 0 & 1 & 3 \\ -1 & 1 & -2 \end{bmatrix}$

註: 本次考試 不可以參考自己的書籍及筆記； 不可以使用字典； 不可以使用計算器

Part II (50%)

1. A zoo wants to set up natural habitats (棲息地) in which to exhibit the following animals, rabbits, tigers, deers(鹿), lions, and owls(貓頭鷹). Unfortunately, some animals will eat some of the others when given the opportunity. Therefore, they cannot be put in the same habitat. Use the graph theory ideas to determine the number of different habitats needed and the placement (安排) of the animals in these habitats. (5%)

	Tiger	Owl	Deer	Lion	Rabbit
Tiger			*	*	*
Owl					*
Deer	*			*	
Lion	*		*		*
Rabbit	*	*		*	

*: animals cannot be placed in the same habitat.

2. Which of the relation on {0, 1} satisfy all the following properties: (1) irreflexive, (2) antisymmetric, (3) asymmetric, and (4) transitive? Also, describe the definition of partial order for a relation on a set. (10%)

3. For $n \geqq 0$, let F_n denote the *n*th Fibonacci number. Prove that $F_0+F_1+\ldots+F_n = F_{n+2} - 1$. (10%)

4. Translate each of these quantifications into English and determine its truth value. (10%)
(a) $\forall x \in R(x^2 \neq -1)$ (b) $\exists x \in R(x^2 = x)$

5. Prove or disprove that the set of odd positive integers is an uncountable set or not. (5%)

6. Solve the recurrence relation $a_n = 3a_{n-1} + 2^n$, with initial condition $a_0 = 2$. (10%)