大同大學 九十二 學年度研究所碩士班入學考試試題

考試科目:基本數學　　　　所別:資訊工程研究所　　　　

註:本次考試 不可以參考自己的書籍及筆記； 不可以使用字典； 不可以使用計算器。

本試題計兩部份：〔一〕離散數學〔二〕線性代數。每部份五十分。請將答案依序寫在答案卷上。

〔第一部份〕離散數學

1. Determine the number of nonnegative integer solutions to the pair of equations. (8%)
   $x_1 + x_2 + x_3 = 6$ and $x_1 + x_2 + x_3 + x_4 + x_5 = 15$, for $x_i \geq 0$, $1 \leq i \leq 5$

2. In the following program segment, *i, j, k*, and *counter* are integer variables. Determine the value that the variable *counter* will have after the segment is executed. (7%)

```
counter := 10
for i := 1 to 15 do
  for j := i to 15 do
    for k := j to 15 do
      counter := counter + 1
```

3. The connective "Nor" or "Not ... or..." is defined for any statements *p, q* by $(p \downarrow q) \equiv \neg (p \vee q)$. Represent the following using only this connective. (9%)
   (a) $\neg p$ (b) $p \vee q$ (c) $p \wedge q$.

4. Let S = {1,2,3,…,29,30}. How many subsets A of S satisfy
   (a) |A| = 5? (2%)
   (b) |A| = 5 and the smallest element in A is 5? (2%)
   (c) |A| = 5 and the smallest element in A is less than 5? (4%)

5. In a certain area of the countryside are four villages. An engineer is to devise a system of two-way roads so that after the system is completed, no village will be isolated. In how many ways can he do this? (8%)

6. Design a finte-state machine according to the given properties. The input is always a bit string.
   (a) Outputs 1 whenever it sees 101; otherwise, outputs 0. (5%)
   (b) Outputs 1 when it sees 101 and thereafter; otherwise, outputs 0. (5%)

<背面繼續>

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<接前頁>

〔第二部份〕線性代數

1. Write the words in the blanks on the answer sheet [2% each]
   - Suppose the product of A and B is the zero matrix: AB = 0. Then the ____(1)____ space of A contains the ____(2)____ space of B. Also the ____(3)____ space of B contains the ____(4)____ space of A.
   - If v_1, v_2, v_3 form a basis of R^3, then the matrix with those three columns is ____(5)____
   - If v_1, v_2, v_3, v_4 span R^3, give all possible ranks for the matrix with those four columns: ____(6)____.

2. Suppose the n by n matrix A_n has 3's along its main diagonal and 2's along the diagonal below and the $(1,n)$ position:

$$A_4 = \begin{bmatrix} 3 & 0 & 0 & 2 \\ 2 & 3 & 0 & 0 \\ 0 & 2 & 3 & 0 \\ 0 & 0 & 2 & 3 \end{bmatrix}$$

   Find by cofactors of row 1 the determinant of A_4 [2%] and then the determinant of A_n for $n > 4$ [2%].

3. P_1: $x + 2y - 3z = 0$ defines a plane in R^3. Find a basis that spans the subspace [4%].
   P_2: $x + 2z = 1$ defines another plane in R^3. Find the general (complete) solution of the intersection of P_1 and P_2 [4%]. Does this intersection define a subspace in R^3? Why or why not? [2%]

4. Let S be the subspace of R^3 spanned by (1 2 2) and (5 4 -2).
   Find an orthonormal basis q_1, q_2 for S by Gram-Schmidt [4%].
   Write down the 3 by 3 matrix P which projects vectors perpendicularly onto S [4%].
   For any vector b in R^3, show that the properties of P lead to the conclusion that Pb is orthogonal to b - Pb [2%].

5. The following matrix is a projection matrix:

$$P = \frac{1}{21}\begin{bmatrix} 1 & 2 & -4 \\ 2 & 4 & -8 \\ -4 & -8 & 16 \end{bmatrix}$$

   (a) What subspace does P project onto? [3%]
   (b) What is the distance from that subspace to b = (1 1 1)? [4%]
   (c) What are the eigenvalues of P? [3%] Is P diagonalizable? If yes, factor the matrix into $P = S\Lambda S^{-1}$ [4%].