

大同大學 九十三年 學年度研究所碩士班入學考試試題

考試科目：基本數學

所別：資訊工程研究所

第2/2頁

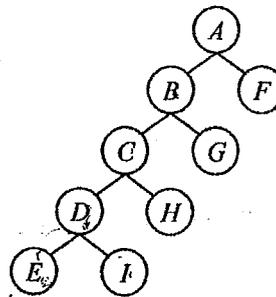
註：本次考試 不可以參考自己的書籍及筆記； 不可以使用字典； 不可以使用計算器。

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離散數學 (計五十分)

1. Six married couples organize a dance party. In how many ways can they be arranged into dancing pairs, when it is required that nobody dances with their own spouse? (5%)
2. How many subsets of $\{a, e, i, o, u, y\}$ contain exactly 4 elements? (2%)
3. Consider the following relations:
 $f = \{(1, 2), (2, 3), (3, 4), (4, 1)\}$
 $g = \{(1, 1), (1, 2), (3, 3), (3, 4)\}$
 $h = \{(1, 4), (2, 3), (3, 3), (4, 2)\}$
 - a) Which of the relations f, g and h is (are) function(s)? (2%)
 - b) Which of the inverse relations f^{-1}, g^{-1} and h^{-1} is (are) function(s)? (2%)
 - c) Which of the relations f, g and h is (are) invertible function(s)? (2%)
4. A relation of R is defined on the integers by $aRb \Leftrightarrow |a - b| \leq 1$.
 - a) Is R reflexive? (2%)
 - b) Is R symmetric? (2%)
 - c) Is R antisymmetric? (2%)
 - d) Is R transitive? (2%)Give reasons of your answers.
5. Let A be a set with n elements and B a set with m elements, with $n \leq m$. How many different one-to-one functions are there with domain A and range B ? (5%)
6. Given the root of a tree as parameter, the following algorithm (scantree) is used to print out the labels of all its tree nodes.

```
Algorithm scantree(treenode root)
if root=nil return;
scantree(root.left);
print root.label;
Scantree(root.right);
```



- Feeding the root of the above tree as the parameter of the algorithm, what string will be printed out? (5%)
7.
 - a) Describe what strings are accepted by the regular expression $a^+(bb)^*$ in English or Chinese (5%).
 - b) Construct a finite state machine to accept the language described by the regular expression in a. (5%)
 8. Use a truth table to determine whether the implication $\neg q \rightarrow \neg [p \wedge (\neg p \vee q)]$ is a tautology or not. (5%)
 9. Solve the following second-order linear recurrence, $a_n = a_{n-1} + 6a_{n-2}$, with initial conditions $a_0 = 1, a_1 = 2$. (4%)

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第 1/2 頁

註：本次考試 不可以參考自己的書籍及筆記； 不可以使用字典； 不可以使用計算器。

本試題分線性代數及離散數學兩部份，兩部份的題目不可穿插作答，否則不予計分。

線性代數 (計五十分)

1. Find an SVD (Singular Value Decomposition) for $A = \begin{bmatrix} 6 & 2 \\ -7 & 6 \end{bmatrix}$. (10%)
2. Consider the dynamical system
$$\bar{x}(t+1) = \begin{bmatrix} 1/2 & 3/4 \\ 1/2 & 1/4 \end{bmatrix} \bar{x}(t),$$
 with initial value $\bar{x}_0 = \begin{bmatrix} 100 \\ 0 \end{bmatrix}$. Find $\lim_{t \rightarrow \infty} \bar{x}(t)$. (10%)
3. (a) If matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, what is the inverse of A ? (4%)
(b) If $A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & -27 \end{bmatrix}$, find A^{-1} . (3%)
(c) If matrix B is similar to matrix A , what is the relationship between the determinants $\det(A)$ and $\det(B)$? (3%)
4. For the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, determine the images and kernels of the matrices A and A^2 . (10%)
5. Fit a quadratic function to the four data points $(a_1, b_1) = (-1, 8)$, $(a_2, b_2) = (0, 8)$, $(a_3, b_3) = (1, 4)$, and $(a_4, b_4) = (2, 16)$. (10%)

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