1. Determine whether $(p \to (q \vee r)) \to ((p \to r) \vee (p \to q))$ is a tautology. [4%]

2. How many ways are there to assign six different jobs to three different employees if the hardest job is assigned to the most experienced employee and the easiest job is assigned to the least experienced employee? [6%]

3. What is wrong with this "proof"? [4%]
Theorem: $a^n = 1$ for all nonnegative integers n, where a is a nonzero real number.
Basic step: $a^0 = 1$ is true by the definition of a^0.
Inductive step: Assume that $a^j = 1$ for all nonnegative integers j with $j \le n$. Then note that

$$a^{k+1} = \frac{a^k \cdot a^k}{a^{k-1}} = \frac{1 \cdot 1}{1} = 1.$$

4. Use mathematical induction to show that 21 divides $4^{n+1} + 5^{2n-1}$ whenever n is a positive integer. [6%]

5. (a) If f and $f \circ g$ are one-to-one, does it follow that g is one-to-one? Explain your answer. [4%]
(b) If f and $f \circ g$ are onto, does it follow that g is onto? Explain your answer. [4%]

6 Find the number of solutions of $x_1 + 2x_2 + x_3 = 17$ where x_i's are nonnegative integers with $2 \le x_1 \le 5, 3 \le x_2 \le 6, 4 \le x_3 \le 7$ [8%]

7. Solve the following recurrence relation [8%].

$$a_n = 5a_{n-1} - 6a_{n-2} + 2^n, a_0 = 1, a_1 = 1$$

8. The Computer Science Department has 5 committees that meet once a month. How many different meeting times must be used to guarantee that no one is scheduled to be at 2 meetings at the same time? The committees and their members are $C_1 = \{$Adams, Brooks, Casey$\}$, $C_2 = \{$Brooks, Casey, Frank$\}$, $C_3 = \{$Brooks, Daly, Ellen$\}$, $C_4 =$

{Ellen, Frank}, C_5 = {Brooks, Casey, Daly} [6%].