大同大學 97 學年度研究所碩士班入學考試試題

考試科目:基本數學　　所別:資訊工程研究所　　

註:本次考試 不可以參考自己的書籍及筆記； 不可以使用字典； 不可以使用計算器。

Part One: Discrete Mathematics [50%]

Mark true(T) or false (F) for Questions 1-2. [4%]

1. The statement $p \to (q \to r)$ is equivalent to $(p \to q) \to r$.
2. The statement $(q \wedge (p \to \neg q)) \to \neg p$ is a tautology.

Multiple Choice Problems (3-9): each problem has *exactly one* correct choice.

3. In the questions below $P(x,y)$ means "$x + 2y = xy$", where x and y are integers. How many of these statements are TRUE? [4%]

$\forall x \exists y P(x,y)$. $\exists x \forall y P(x,y)$. $\forall y \exists x P(x,y)$. $\exists y \forall x P(x,y)$.

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4.

4. In the questions below, suppose $A = \{a,b,c\}$ and $B = \{b,\{c\}\}$. How many of these statements are TRUE ? [4%]

$c \in A - B$. $|P(A \times B)| = 64$. $\{c\} \subseteq B$.
$\{a,b\} \in A \times A$. $\{b,\{c\}\} \in P(B)$. $\{\{\{c\}\}\} \subseteq P(B)$.

(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

In the questions below suppose the variable x represents students, $F(x)$ means "x is a freshman", and $M(x)$ means "x is a math major". Match the statement in symbols with one of the English statements in this list: [8%]

a. Some freshmen are math majors.
b. Every math major is a freshman.
c. No math major is a freshman.

5. $\neg \forall x(F(x) \to \neg M(x))$.

6. $\forall x(M(x) \to \neg F(x))$.

7. $\forall x(M(x) \to F(x))$.

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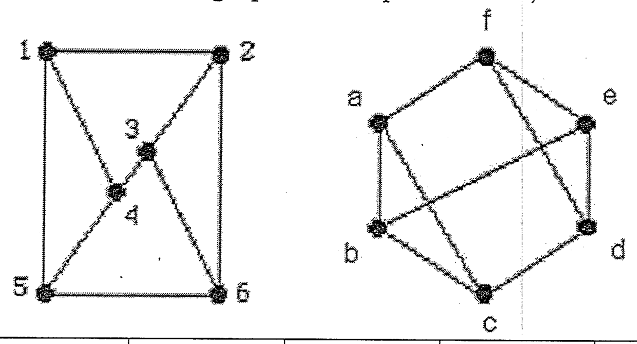
8. $\exists x(F(x) \wedge M(x))$.

Answer these questions in sufficient details.

9. Are these two graphs isomorphic? If so, label the corresponding vertex pairs. [6%]



1	2	3	4	5	6

In the expansion of $(1+x^2+x^4)^5$:

10. How many terms are there? [4%]

11. What is the coefficient of x^8 ? [2%]

The set $S = \{1,2,3,\ldots,10\}$. Find the number of subsets of S satisfying each of the following conditions:

12. ... contains no odd numbers [2%]

13. ... contains exactly three elements, all of them even. [2%]

14. ... contains exactly five elements, the sum of which is even. [2%]

15. Solve the following recurrence relation: [6%]

$a_n = 2a_{n-1} + 5, \quad a_0 = 3.$

16. How many different channels are needed for six television stations (A,B,C,D,E,F) whose distances (in miles) from each other are shown in the following table? Assume that two stations cannot use the same channel when they are within 150 miles of each other? [6%]

	A	B	C	D	E	F
A	–		175	100	50	100
B	85	–	125	175	100	130
C	175	125	–	100	200	250
D	100	175	100	–	210	220
E	50	100	200	210	–	100
F	100	130	250	220	100	–

Part II Linear Algebra [50 points]

True/False Problems [1-10]: Mark true (T) or false (F) for each of the statement below.
[2 points each, with a penalty of −1 point for each wrong answer.]

1. The subset of vectors in R^3 with $b_1b_2b_3 = 0$ forms a subspace.
2. The subset of vectors in R^3 with $b_1 + b_2 + b_3 = 0$ forms a subspace.
3. The block matrix $\begin{bmatrix} 0 & A \\ A & 0 \end{bmatrix}$ is symmetric (where A is a matrix).
4. If w_1, w_2, w_3 are independent vectors, the sums $v_1 = w_2 + w_3$, $v_2 = w_1 + w_3$, $v_3 = w_1 + w_2$ are independent.
5. A and A^T have the same nullspace.
6. If $AB = B$ then $A = I$.
7. If A and B share the same four subspaces then A is a multiple of B.
8. If the eigenvalues of A are 2,2,5 then the matrix is certainly not diagonalizable.
9. If the only eigenvectors of A are multiples of (1,4) then A has no diagonalization of $S\Lambda S^{-1}$
10. If the row space equals the column space then $A^T = A$.

11. [5%] Compute the determinant of A: $A = \begin{bmatrix} 101 & 201 & 301 \\ 102 & 202 & 302 \\ 103 & 203 & 303 \end{bmatrix}$.

12. [5%] Find the complete solution (also called the general solution) to

$$\begin{bmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}.$$

13. [8%] For the following 4 by 4 checkerboard matrix C:

$$C = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

(i)Find a basis for the nullspace of C.
(ii)Find the eigenvalues of C.

14. For the following 3 by 2 matrix A:

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ -2 & 4 \end{bmatrix}$$

(i) [6%] Find orthonormal vectors q_1, q_2, q_3 such that q_1, q_2 span the column space of A.
(ii) [2%] Which of the four fundamental subspaces of A contains q_3?
(iii) [4%] Solve Ax =(1,2,7) by least square.