

大同大學 99 學年度研究所碩士班入學考試試題

考試科目：基本數學

所別：資訊工程研究所

第 1/4 頁

註：本次考試 不可以參考自己的書籍及筆記； 不可以使用字典； 不可以使用計算器。

PART I. Linear Algebra [50 points]

True/False Problems [1-10, 20%]: Mark true (T) or false (F) for each of the statement below.
[2 points each, with a penalty of -1 point for every wrong answer.]

1. A block matrix $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is symmetric if A, B, C, D are all symmetric.
2. $(1,1,1)$ is perpendicular to $(1,1,-2)$ so the planes $x+y+z=0$ and $x+y-2z=0$ are orthogonal subspaces.
3. If all the cofactors of A are zero, A has no inverse!
4. If none of the cofactors of A are zero, A is sure to be invertible.
5. A matrix whose eigenvalues are all zeros is a zero matrix.
6. If A and B are symmetric, then $(A+B)(A-B)$ is also symmetric.
7. If A and B are symmetric, then $A+B$ is also symmetric.
8. If A and B are positive definite, then $A+B$ is also positive definite.
9. A symmetric matrix can't be similar to a nonsymmetric matrix.
10. $(1,1,2)$, $(2,2,3)$ and $(3,3,8)$ can be a basis for \mathbb{R}^3 .

11. What is the closest point on the plane $x-2y-z=0$ to $(1,3,1)$ [6%]? Find the shortest distance [2%].

12. [5%] Compute the determinant:

$$\det \begin{bmatrix} 1 & 2 & 0 & 3 & -1 \\ 1 & 0 & 0 & 2 & 0 \\ 3 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 1 \end{bmatrix}.$$

13. [9%] Find the eigenvalues and eigenvectors of $I - P$, where $P = uu^T$, $u = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$.

14. A transformation is defined by $L \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} = \begin{pmatrix} u_1 + u_3 + u_4 \\ u_2 + 2u_3 \\ -u_2 + u_4 \end{pmatrix}$.

- Find the standard matrix representing L . [2%]
- Find the kernel of L . Is L one-to-one? [3%]
- Find the range of L . Is L onto? [3%]

<背面繼續>

大同大學 99 學年度研究所碩士班入學考試試題

考試科目：基本數學

所別：資訊工程研究所

第 2/4 頁

註：本次考試 不可以參考自己的書籍及筆記； 不可以使用字典； 不可以使用計算器。

< 接前頁 >

Part II. Discrete Math [50 Points]

True (T) or False (F) [20%]:

2 points for each correct answer, and -1 point for each wrong answer. Be careful.

1. If $a \equiv b \pmod{m}$, and $a \equiv c \pmod{m}$, then $a \equiv b + c \pmod{m}$.
2. If $a \equiv b \pmod{m}$, then $2a \equiv 2b \pmod{m}$.
3. $\gcd(n+1, 2n+1) = 1$.
4. If the well-formed formula ϕ is a tautology then ϕ is satisfiable.
5. $(\neg q \wedge (\neg p \rightarrow q)) \rightarrow \neg p$ is a tautology.
6. Given two sets A and B , if A and $A \cap B$ are countably infinite, then B must be countably infinite.
7. No function can be both $O(n^2)$ and $O(n^3)$.
8. Let w be a string of six letters from $\{a, b, c\}$ where each letter is chosen independently with probability $1/3$ of each letter. Then the probability that w has exactly two occurrences of each letter is less than 5%.
9.
$$\binom{100}{0} + \binom{100}{2} + \binom{100}{4} + \cdots + \binom{100}{100} = \binom{100}{1} + \binom{100}{3} + \binom{100}{5} + \cdots + \binom{100}{99}.$$
10. Let A , B , and C be three *pairwise independent* random variables over the same event space. (This means that A and B , A and C , and B and C are each independent pairs.) Then $\text{Prob}(A|B) = \text{Prob}(A|C)$.

Multiple Choice [30%]:

Each of the following questions has exactly one correct choice. 2 points for each correct choice, and -0.5 point for each wrong choice. Be careful.

1. How many integral solutions are there of $x_1 + x_2 + x_3 + x_4 + x_5 = 30$ where $x_i \geq 0$

(a) $\binom{36}{30}$;

(b) $\binom{35}{30}$;

(c) $\binom{34}{30}$;

(d) $\binom{34}{29}$;

(e) none of the above.

2. Answer problem 1 for $x_i \geq 1$

(a) $\binom{29}{25}$;

(b) $\binom{30}{25}$;

(c) $\binom{29}{24}$;

(d) $\binom{30}{26}$;

(e) none of the above.

大同大學 99 學年度研究所碩士班入學考試試題

考試科目：基本數學

所別：資訊工程研究所

第 3/4 頁

註：本次考試 不可以參考自己的書籍及筆記； 不可以使用字典； 不可以使用計算器。

3. Which of the following statements is true?
- (a) If f_1 and f_2 are $O(g)$, then $f_1 f_2$ is $O(g)$.
 - (b) If f is $O(g)$, then f is $O(g/2)$.
 - (c) If f is $O(g)$, then g is $O(f)$.
 - (d) If f is $O(g)$, then g is not $O(f)$.
 - (e) none of the above.
4. According to De Morgan's laws, $\overline{A \cup (B \cap C)} = ?$
- (a) $\bar{A} \cap (B \cap C)$;
 - (b) $\bar{A} \cup (\bar{B} \cap \bar{C})$;
 - (c) $\bar{A} \cup (B \cup C)$;
 - (d) $\bar{A} \cap (\bar{B} \cup \bar{C})$;
 - (e) none of the above.
5. Suppose that A, B, C are sets with $|A| = 24, |B| = 23, |C| = 22, |A \cap B| = 9, |A \cap C| = 7, |A \cap B \cap C| = 4$, and $|A \cup B \cup C| = 49$. What is $|B - C|$?
- (a) 9;
 - (b) 15;
 - (c) 7;
 - (d) 13;
 - (e) none of the above.
6. Suppose $f: R \rightarrow R$ has the following property for all real numbers x and y : if $x < y$ then $f(x) < f(y)$, i.e., f is strictly increasing. Which of the following is true?
- (a) f must be both 1-1 and onto R .
 - (b) f is not necessary 1-1 and not necessary onto R .
 - (c) f must be both 1-1 but is not necessary onto R .
 - (d) f is onto R but is not necessary 1-1.
 - (e) none of the above.
7. What is the minimum number of persons in a group necessary to guarantee that at least 3 of them were born on the same day of the week?
- (a) 15;
 - (b) 9;
 - (c) 4;
 - (d) 21;
 - (e) none of the above.
8. Define the function $f(n)$ by $f(1) = 1$ and $f(n) = n + f(n-1)$ for $n > 1$. What is a non-inductive representation for $f(n)$?
- (a) $f(n) = n+1$;
 - (b) $f(n) = n$;
 - (c) $f(n) = 2n-1$;
 - (d) $f(n) = n(n+1)/2$;
 - (e) none of the above.
9. Which of the following distributions is not discrete:
- (a) Gamma distribution;
 - (b) Poisson distribution;
 - (c) Geometric distribution;
 - (d) binomial distribution;
 - (e) none of the above.

<背面繼續>

大同大學 99 學年度研究所碩士班入學考試試題

考試科目：基本數學

所別：資訊工程研究所

第 4/4 頁

註：本次考試 不可以參考自己的書籍及筆記； 不可以使用字典； 不可以使用計算器。

<接前頁>

10. Which of the following distributions is memoryless:

- (a) Gamma distribution;
- (b) Poisson distribution;
- (c) Geometric distribution;
- (d) binomial distribution;
- (e) none of the above.

11. The probability of a binomially distributed random variable can be approximated using a Poisson distribution if

- (a) n is very small and p very small;
- (b) n is very large and p very small;
- (c) n is very small and p very large;
- (d) n is very large and p very large;
- (e) none of the above.

12. How many binary sequences of length 8 begin with 11 and do not end with 111?

- (a) 63;
- (b) 42;
- (c) 36;
- (d) 56;
- (e) none of the above.

13. Let $|A|=3$ and $|B|=4$. The number of function $f: A \rightarrow B$ is:

- (a) 3^4 ;
- (b) 4^3 ;
- (c) 2^7 ;
- (d) 2^{12} ;
- (e) none of the above.

14. Let $|A|=3$ and $|B|=4$. The number of relations from A to B is

- (a) 3^4 ;
- (b) 4^3 ;
- (c) 2^7 ;
- (d) 2^{12} ;
- (e) none of the above.

15. Let $|A|=3$ and $|B|=4$. The number of 1-1 function $f: A \rightarrow B$ is

- (a) 6;
- (b) 12;
- (c) 24;
- (d) 48;
- (e) none of the above.