大同工學院 87 學年度研究所招生入學考試試題

考試科目 工程數學　　所別 通訊工程研究所

註：本次考試 [不可] 參考自己的書籍及筆記 [不可] 查字典 [不可] 使用計算器

1. (a) Find the least squares approximation of $\sin \pi x$ over the interval [-1, 1] by a polynomial of the form $a_0 + a_1 x + a_2 x^2$.

   (b) Find the mean square error of the approximation. (20%)

2. Solve the following system of linear equations

$$\begin{cases} 3x_1 + 2x_2 + 3x_3 - 2x_4 = 1 \\ x_1 + x_2 + x_3 = 3 \\ x_1 + 2x_2 + x_3 - x_4 = 2 \end{cases}$$

(10%)

3. Let L be the line $y = 4x$, and let T be the reflection of R^2 about L, where $R^2 \equiv \{(a,b): a,b \in R\}$.

   (a) Find an expression for $T(x,y)$.

   (b) Find an expression for $T^{-1}(x,y)$, the inverse of T.

   (c) Find all the eigenvalues of T.

   (d) Find an expression for $T^m(x,y)$, where

$$T^m(x,y) \equiv T^{m-1}(T(x,y)).$$

(20%)

4. Suppose that the random variable X is continuous. Let the probability density function (pdf) be given by

$$f(x) = \begin{cases} ax & 0 < x < 1 \\ 0 & eleswhere \end{cases}$$

   (a) Find the constant a

   (b) Evaluate the conditional probability $P(X \le \frac{1}{2} \mid \frac{1}{3} \le X \le \frac{2}{3})$

(10%)

5. Suppose that X has pdf

$$f(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & eleswhere \end{cases}$$

   Let $H(x) = 3x + 1$

   (a) Find the cumulative distribution function $G(y)$ of $Y = H(X)$.

   (b) Find the pdf of Y.

(10%)

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6. Assume that the two-dimensional continuous random variable (X,Y) has joint pdf given by

$$f(x,y)=\begin{cases} x^2+\dfrac{xy}{3} & 0\le x\le 1;\ 0\le y\le 2 \\ 0 & eleswhere \end{cases}$$

Please compute $P(B)$ for $B=\{X+Y\ge 1\}$.

(10%)

7. X is a continuous random variable with the following pdf:

$$f(x)=\begin{cases} \dfrac{e^{x}}{2} & if\ x\ge 0 \\ \dfrac{e^{-x}}{2} & if\ x<0 \end{cases}$$

Let $Y=|X|$, then find $E(Y)$.

(10%)

8. The density function of the normal random variable X with mean μ and variance σ^2 is denoted as $n(x;\mu,\sigma)$.

(a) Please write the mathematic expression of $n(x;\mu,\sigma)$.

(b) The curve of $n(x;\mu,\sigma)$ has a maximum which occurs at $x=?$

(c)The curve of $n(x;\mu,\sigma)$ is symmetric about a vertical axis through $x=?$

(d)The total area under the curve of $n(x;4,5)$ and above the horizontal axis and right to the vertical line $x=4$ is equal to A. What is the value of A?

(10%)