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大同工學院 八十八 學年度研究所招生入學考試試題

考試科目:工程數學　　　　所別:通訊工程研究所　　　　第1/1頁

註:本次考試不可以參考自己的書籍及筆記；不可以使用字典；　可以使用計算器。

1. Let X be the input to a communication channel and let Y be the output. The input to the channel is +1 volt or −1 volt with equal probability. The output of the channel is the input plus a noise noise N that is uniformly distributed in the interval −2 volts to +2 volts.
   (a) Find the probability that Y is negative given that X is +1. (10%)
   (b) Find $P(X=+1, Y\le 0)$. (5%)

2. Two packet streams arrive at a switching node; one from a voice source and the other from a data source. Let X be the number of time slots until a voice packet arrives and Y the number of time slots till a data packet arrives. If X and Y are geometrically distributed with parameters p and q respectively, find the distribution of the time, in terms of time slots, till a packet arrive at the node. (15%)

3. Consider a packet stream whereby packets arrive according to a Poisson process with rate 100 packets/sec. If the interarrival time between any two packets is less than the transmission time of the first to arrive, the two packets are said to collide. Find the probability that a packet does not collide with either its predecessor or its successor assuming:
   (a) All packets have a transmission time of 2 msec. (10%)
   (b) Packets have independent, exponentially distributed transmission times with mean 2 msec. (10%)

4. For any $A, B \in M_{2x2}(F)$, where $M_{2x2}(F)$ is the set of all 2x2 matrices with entries from a field F, prove that $\det(AB)=\det(A)\det(B)$. (10%)

5. Let $P(R)$ be the set of all polynomials with coefficients from the field R, and $P_2(R)$ consist of all polynomials in $P(R)$ having degree less than or equal to 2. Let $\beta = \{1, x, x^2\}$ and β is the basis for $P_2(R)$. Let T: $P_2(R) \to P_2(R)$ be defined by $T(f) = 2f'' - f' + 2f$, where f' and f'' denote the first and second derivatives of f with respect to x. Compute T^{-1} if it exists. (15%)

6. $A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$.
   (a) Find all the eigenvectors of the matrix A. (6%)
   (b) Test A for diagonalizability, and if A is diagonalizable, find a matrix Q such that $Q^{-1}AQ$ is a diagonal matrix. (4%)

7. $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$.
   (a) Find A^{-1}. (3%)
   (b) Compute $2A^8 - 14A^7 - 4A^6 + A^5 - 4A^4 - 21A^3 - 19A^2 - 6A + I$. (12%)