1. List the following statements as being true or false. (20%)
   (a) If a vector space has a finite basis, then the number of vectors in every basis is the same.
   (b) A matrix must be square in order to possess an inverse.
   (c) The sum of two $n \times n$ elementary matrices is an elementary matrix.
   (d) Elementary row operations preserve rank.
   (e) Any homogeneous system of linear equations has at least one solution.
   (f) If B is the matrix obtained by interchanging the rows of a 2×2 matrix A, then $\det(B) = -\det(A)$.
   (g) If E is an elementary matrix, then $\det(E) = \pm 1$.
   (h) A matrix M is invertible if and only if $\det(M) = 0$.
   (i) Eigenvalues must be non-zero scalars.
   (j) Eigenvectors corresponding to the same eigenvalue are always linearly dependent.

2. Solve the matrix equation $AX = B$, where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 1 & 1 & -1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \qquad (10\%)$$

3. Find the eigenvalues of the matrix and determine the dimensions of the corresponding eigenspaces.

$$A = \begin{pmatrix} 1 & -2 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & -2 & 1 \end{pmatrix} \qquad (10\%)$$

4. Show that the following function defines an inner product on R^2.
$$\langle x, y \rangle = a_1 b_1 + c a_2 b_2$$
where $x = (a_1, a_2)$, $y = (b_1, b_2)$, and c is a positive constant. (10%)

5. A random variable X has a mean 8, a variance 9, and an unknown probability distribution. Find an upper bound of $P(|X-8| \geq 6)$. (10%)

6. The length of time for one customer to be served at a cafeteria is a random variable having an exponential distribution with a mean of 4 minutes. What is the probability that a person is served in less than 2 minutes? (10%)

7. At busy time a telephone switch is very near capacity, so people cannot find a line to use. Suppose that the probability of a success connection is 0.1 during busy time. What is the probability that 3 attempts are necessary for a successful call? (10%)

8. If the number of telephone calls received by a service center is a Poisson random variable with an average of 5 calls coming per minute. What is the probability that fewer than 2 calls come in any minute? (10%)

9. Let X and Y have the joint probability mass function q((1,1))=1/12, q((2,1))=1/6, q((3,1))=1/12, q((1,2))=1/6, q((2,2))=1/3, q((3,2))=1/6. Find $E(X)$ and $E(Y)$. (10%)