大同大學 九十學年度研究所招生入學考試試題

考試科目:工程數學　　所別: 通訊工程研究所　　

註:本次考試不可以參考自己的書籍及筆記； 不可以使用字典； 不可以使用計算器。

1. Label the following statements as being TRUE or FALSE. (20%)
   (a) The inverse of a triangular matrix is still triangular.
   (b) The rank of a matrix is equal to the number of its non-zero columns.
   (c) rank $(A+B) = \text{rank}(A) + \text{rank}(B)$, where A and B are both $m\times n$ matrices
   (d) $\det(AB)= \det(A)\cdot\det(B)$, where A and B are both $n\times n$ matrices.
   (e) If A is a square matrix of order n, then $\det(A) = -\det(A^{T})$.
   (f) Similar matrices always have the same eigenvectors.
   (g) The fact that an $n\times n$ matrix A has n distinct eigenvalues does not gurantee that A is diagonalizable.
   (h) Every matrix is similar to its Jordan canonical form.
   (i) If V and W are orthogonal subspaces of R^n, then their intersection is an empty set.
   (j) Every orthogonal set is linearly independent.

2. Find the eigenvalues of the following matrix. Then find a matrix P such that $P^{-1}AP$. (10%)

$$A=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 5 & -10\\ 1 & 0 & 2 & 0\\ 1 & 0 & 0 & 3\end{bmatrix}$$

3. Let X be the input to a communication channel and let Y be the output. The input to the channel is +1 volt or −1 volt with equal probability. The output of the channel is the input plus a noise noise N that is uniformly distributed in the interval −3 volts to +3 volts. Find the probability that Y is negative given that X is +1. (10%)

4. A computer system crashes 10 times a year on average and the time to the next crash is memoryless. Given that there was exactly one crash in May, what is the probability that it occurred on 11 May? (10%)

5. Find the distribution function of the minimum of a finite set of independent random variables $\{X_1, X_2, \ldots, X_n\}$, where X_i has distribution function F_{X_i}. (10%)

6. Consider the function $h(x) = (x-½)^3$ and let X be a Bernoulli random variable with parameter p. (10%)
   (a) Find $E[h(X)]$ and $h(E[X])$.
   (b) Determine the condition that makes the relationship, $E[h(X)] = h(E[X])$, hold.

7. Consider the sequences $x_1[n] = \{0, 1, 2, 3, 4\}$, $x_2[n] = \{0, 1, 0, 0, 0\}$ and their five-point Discrete Fourier Transform $X_1(k), X_2(k)$. (15%)
   (a) Determine a sequence of $y[n]$ so that $Y(k) = X_1(k)\, X_2(k)$
   (b) Let $v[n] = \{3, 4, 0, 1, 2\}$ where n ranges from −1 to 3, i.e., $v[-1] = 3$, etc. What is the relation between $X_1(k)$ and the Fourier transform of $v[n]$, $V(\omega)$?

8. Let $x(t)=\dfrac{\sin(\pi 2400t)}{\pi 3600t}$ (15%)
   (a) Determine the Fourier transform of $x(t)$.
   (b) Determine the Nyquist rate of $x(t)$.