大同大學 九十一 學年度研究所招生入學考試試題

考試科目:工程數學　　　所別: 通訊工程研究所　　　第1/1頁

註:本次考試不可以參考自己的書籍及筆記； 不可以使用字典； 不可以使用計算器。

1.[10 points] Find $(I\text{-}A)^3$ and A^{-1} where $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

2.[10 points] Suppose A is a 5 by 3 matrix with orthonormal columns. Evaluate the determinants, $\det(A^TA)$ and $\det(AA^T)$.

3.[10 points] Each of the following independent questions refers to the matrix $A = \begin{bmatrix} 4 & 1 \\ x & -4 \end{bmatrix}$.

(a) Give a value of x such that 2 is one of the eigenvalues of A.
(b) Give a value of x such that A has some repeated eigenvalues.

4.[10 points] A person is given two questions and must decide which question to answer first. Question A will be answered correctly with probability 0.8, and the person will then receive as prize $100, while question B will be answered correctly with probability 0.5, and the person will then receive as prize $200. If the first question attempted is answered incorrectly, the quiz terminates, i.e., the person is not allowed to attempt the second question. If the first question is answered correctly, the person is allowed to attempt the second question. Which question should be answered first to maximize the expected value of the total prize money received? Justify your answer.

5.[10 points] Suppose $P(X=0)=0.5$, $P(X=1)=0.3$, and $P(X=2)=0.2$.
(a) Find the moment-generating function $M_X(t)$.
(b) Use $M_X(t)$ to find $E[X]$.

6.[10 points] You are allowed to take a certain test three times, and your final score will be the maximum of the test scores. Assume that your socre in each test takes one of the integer values from 1 to 10 with equal probability, independently of the scores in other tests. What is the probability mass function of your final score?

7.[10 points] Suppose you and your advisor have an appointment at a given time, and each, independently, will be late by an amount of time that is exponentially distributed with parameter λ. What is the probability density function of the difference between your times of arrival?

8.[10 points] $$x(t) = \frac{\sin(7200\pi t)}{1800\pi t}$$

$\frac{\pi}{\lambda}[\delta(w-m)-\delta(w+m)]$

(a) Compute the Fourier transform of $x(t)$.
(b) What is the Nyquist sampling rate of $x(t)$?

9.[10 points] Determine the Fourier transform of $y(t)$ in terms of the Fourier transform of $x(t)$ and real constant f_o where

$$y(t) = \text{Real } \{x(t)\exp(j2\pi f_o t)\}.$$

10.[10 points] Let $x(t)$ and $y(t)$ be two periodic signals with period T_o. If x_n and y_n represent the Fourier series coefficients of these two signals. Show that

$$\int_{\alpha}^{\alpha+T_o} x(t)y^*(t)\,dt = T_o \sum_{n=-\infty}^{\infty} x_n y_n^*$$

where "*" denotes the complex conjugate.