

大同大學 九十二 學年度研究所碩士班入學考試題

考試科目：工程數學

所別：通訊工程研究所

第1頁 共1頁

註：本次考試不可以參考自己的書籍及筆記； 不可以使用字典； 不可以使用計算器。

1. Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & 0 & 0 \end{bmatrix}$, $a \neq 0$. Find A^3 and A^{-1} . (10%)

2. Evaluate the determinant of $\begin{bmatrix} 1 & 1 & 2 & 1 \\ 3 & 1 & 4 & 5 \\ 7 & 6 & 1 & 4 \\ 1 & 1 & 3 & 3 \end{bmatrix}$. (10%)

3. A system can be written as $X_{n+1} = AX_n + B$ where $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$. Solve the system given that $X_0 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$. (10%)

4. A committee consists of three men and four women. A subcommittee of three members is chosen at random. What is the probability that of a male majority on the subcommittee? (10%)

5. On a bus stop, on average 4.8 buses pass every hour. Assume the waiting time until the next bus passes has an exponential distribution. For any $t > 0$, let X_t be the number of buses that pass between 6:00 and t hours later. (20%)

(a) Let Y be the number of buses that pass between 7:30 and 8:30. Express Y as a function of $X_{1.5}$, X_2 , and $X_{2.5}$.

(b) Find $E[Y]$ and $\text{Var}(Y)$.

(c) Let Z be the number of buses that pass between 6:00 and 8:00. Find $\text{Cov}(Y, Z)$.

6. Let X be the bigger of the two numbers that turn up when two ordinary dice are rolled. Find $E[X]$ and $\text{Var}(X)$. (10%)

7. A periodic function can be expressed over one period as follows:

$$f(t) = \begin{cases} A & \text{for } -\frac{d}{2} < t < \frac{d}{2} \\ 0 & \text{for } -\frac{T}{2} < t < -\frac{d}{2}, \frac{d}{2} < t < \frac{T}{2} \end{cases}$$

(a) The period of the function is T . Sketch the periodic function $f(t)$. (5%)

(b) Find the frequency spectra for $f(t)$. (10%)

(c) Sketch the amplitude spectrum for $f(t)$ when $d = \frac{1}{8}$ and $T = \frac{1}{2}$ seconds. (5%)

8. Find the inverse Fourier transforms of $\frac{1}{1+j\omega}$ and $\frac{1}{(1+j\omega)^2}$. (10%)