

# 大同大學 96 學年度研究所碩士班入學考試試題

考試科目：工程數學

所別：通訊工程研究所

第 全 頁

註：本次考試 不可以參考自己的書籍及筆記； 不可以使用字典； 不可以使用計算器

1. [15 points] Let  $A = \begin{bmatrix} 5 & -8 \\ 3 & -5 \end{bmatrix}$ 
  - (1) Find the eigenvalues of  $A$ .
  - (2) Find an eigenvector for each eigenvalue of  $A$ .
2. [20 points] The determinant of a matrix  $A$  is denoted as  $\det A$ . Answer the following questions:
  - (1) What is the determinant of the transpose of  $A$ ?
  - (2) If two rows of  $A$  are equal, what is value of  $\det A$ ?
  - (3) If two rows of  $A$  are interchanged, what is the determinant?
  - (4) If two columns of  $A$  are interchanged, what is the determinant?
3. [15 points] Two random variables  $X$  and  $Y$  are independent and uniformly distributed in the interval  $[0,1]$ . Give the probability density function of  $W = X + Y$ .
4. [20 points] A telephone company charges for calls in the following way: 3 dollars for the first 5 minutes or less; 1 dollar per minute for any additional time.
  - (1) Thus, if  $X$  is the duration of a call, express the cost  $Y$  as a function of  $X$ .
  - (2) Find the expected value of the cost of a call, assuming that the duration of a call is exponentially distributed with a mean of 3 minutes.
5. [20 points] Suppose  $f(t) = e^{-at}$  for some  $a > 0$  when  $t \geq 0$  and  $f(t)$  is 0 when  $t < 0$ .
  - (1) Find the Fourier transform of  $f(t)$ .
  - (2) Find the Fourier transform of  $f'(t) - 2f''(t) + f(t-3)$ .
6. [10 points]  $F, G$  are the Fourier transforms of the functions  $f, g$  on  $\mathfrak{R}$ . Show that

$$\int_{-\infty}^{\infty} f(t)g(t)dt = \int_{-\infty}^{\infty} F(s)G(-s)ds$$