大同大學九十學年度研究所碩士班入學考試試題

考試科目 ： **微積分**　　所別 ： **事業經營研究所**　　第 1 /2 頁

註 ： 1. 本次考試不可參考書籍及筆記；不可使用字典及計算器。
　　2. 試題紙有兩頁，共 10 題，每題 10 分。

1. Find the relative extrema of $f(x) = (x-1)\cosh x - \sinh x$.

2. Maximum $x_1 x_2 \ldots x_n$

$$x_1 + x_2 + \ldots + x_n = k, \quad k \text{ is a constant value.}$$

Use this result to prove that

$$(x_1 x_2 \ldots x_n)^{1/n} \leqq (\underbrace{x_1 + x_2 + \ldots + x_n}_{k}) / n.$$

3. The Gamma Function $\Gamma(n)$ is defined by

$$\Gamma(n) = \int x^{n-1} e^{-x}\, dx, \quad n > 0, \quad 0 < x < \infty.$$

a. Show that $\Gamma(n+1) = n\Gamma(n)$.
b. Express $\Gamma(n)$ in terms of factorial notation where n is a positive integer.

4. When an employee receives a paycheck at the end of each month, P dollars is invested in a retirement account. These deposits are made each month for t years, and the account earns interest at the annual percentage rate r. If the interest is compounded(複利計息) monthly, the amount A in the account at the end of t years is

$$A = P + P(1 + r/12) + \ldots + P(1 + r/12)^{12t-1}$$

$$= P(12/r)[(1 + r/12)^{12t} - 1].$$

If the interest is compounded continuously, the amount A in the account after t years is

$$A = P + Pe^{r/12} + Pe^{2r/12} + Pe^{3r/12} + \ldots + Pe^{(12t-1)r/12}$$

$$= P(e^{rt} - 1)/(e^{r/12} - 1).$$

Very that each of the sums given above equals the right-hand member of the respective equation.

(Be continued)

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5. Find the power series for $\exp(-x^2)$ and then evaluate the definite integral

$$\int \exp(-x^2)\,dx, \quad 0<x<1,$$

with an error of less than 0.01.

6. Rotate the axes counterclockwise to eliminate the xy-term. Sketch the graph of the resulting equation, showing both sets of axes.

$$7x^2-6\sqrt{3}\,xy+13y^2-16=0.$$

7. Find the angle between vectors **u** and **v** and a unit vector that is orthogonal to both vectors

$$\mathbf{u}=[1 \quad -4 \quad 1] \text{ and } \mathbf{v}=[2 \quad 3 \quad 0].$$

8. Find the relative extrema of $f(x, y)=-x^3+4xy-2y^2+1$.

9. Find the area of the region R that lies below the curve

$$y=4x-x^2$$

above the x-axis and the line

$$y=-3x+6.$$

10. Solve the differential equation

$$y''-10y'+25y=5+6e^x.$$