

# 大同大學九十學年度研究所碩士班入學考試試題

考試科目：微積分 所別：事業經營研究所 第 1/2 頁

註：1. 本次考試不可參考書籍及筆記；不可使用字典及計算器。  
2. 試題紙有兩頁，共 10 題，每題 10 分。

1. Find the relative extrema of  $f(x) = (x-1)\cosh x - \sinh x$ .

2. Maximum  $X_1 X_2 \dots X_n$

$$x_1 + x_2 + \dots + x_n = k, \quad k \text{ is a constant value.}$$

Use this result to prove that

$$(x_1 x_2 \dots x_n)^{1/n} \leq (x_1 + x_2 + \dots + x_n) / n.$$

3. The Gamma Function  $\Gamma(n)$  is defined by

$$\Gamma(n) = \int_0^{\infty} x^{n-1} e^{-x} dx, \quad n > 0, \quad 0 < x < \infty.$$

a. Show that  $\Gamma(n+1) = n\Gamma(n)$ .

b. Express  $\Gamma(n)$  in terms of factorial notation where  $n$  is a positive integer.

4. When an employee receives a paycheck at the end of each month,  $P$  dollars is invested in a retirement account. These deposits are made each month for  $t$  years, and the account earns interest at the annual percentage rate  $r$ . If the interest is compounded (複利計息) monthly, the amount  $A$  in the account at the end of  $t$  years is

$$\begin{aligned} A &= P + P(1 + r/12) + \dots + P(1 + r/12)^{12t-1} \\ &= P(12/r)[(1 + r/12)^{12t} - 1]. \end{aligned}$$

If the interest is compounded continuously, the amount  $A$  in the account after  $t$  years is

$$\begin{aligned} A &= P + Pe^{r/12} + Pe^{2r/12} + Pe^{3r/12} + \dots + Pe^{(12t-1)r/12} \\ &= P(e^{rt} - 1) / (e^{r/12} - 1). \end{aligned}$$

Verify that each of the sums given above equals the right-hand member of the respective equation.

( Be continued)

# 大同大學九十學年度研究所碩士班入學考試試題

考試科目：微積分 所別：事業經營研究所 第 2/2 頁

註：1. 本次考試不可參考書籍及筆記；不可使用字典及計算器。  
2. 試題紙有兩頁，共 10 題，每題 10 分。

5. Find the power series for  $\exp(-x^2)$  and then evaluate the definite integral

$$\int \exp(-x^2) dx, \quad 0 < x < 1,$$

with an error of less than 0.01.

6. Rotate the axes counterclockwise to eliminate the  $xy$ -term. Sketch the graph of the resulting equation, showing both sets of axes.

$$7x^2 - 6\sqrt{3}xy + 13y^2 - 16 = 0.$$

7. Find the angle between vectors  $\mathbf{u}$  and  $\mathbf{v}$  and a unit vector that is orthogonal to both vectors

$$\mathbf{u} = [1 \quad -4 \quad 1] \text{ and } \mathbf{v} = [2 \quad 3 \quad 0].$$

8. Find the relative extrema of  $f(x, y) = -x^3 + 4xy - 2y^2 + 1$ .

9. Find the area of the region  $R$  that lies below the curve

$$y = 4x - x^2$$

above the  $x$ -axis and the line

$$y = -3x + 6.$$

10. Solve the differential equation

$$y'' - 10y' + 25y = 5 + 6e^x.$$

Handwritten calculation for problem 6:

$$-\left(\frac{4}{3}\right)^3 + 4\left(\frac{4}{3}\right)\left(\frac{4}{3}\right) - 2\left(\frac{4}{3}\right)^2 + 1$$

$$= -\frac{64}{27} + \frac{64}{9} - \frac{32}{9} + 1$$

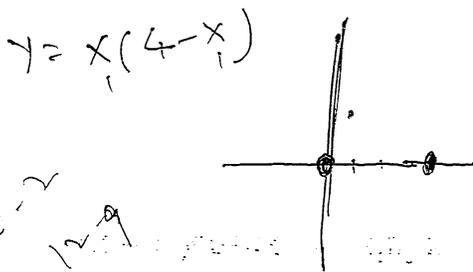
$$= \frac{96}{32} - \frac{64}{32} + \frac{32}{32} + \frac{32}{32} = \frac{96 - 64 + 32 + 32}{32} = \frac{96}{32} = 3$$

Handwritten calculation for problem 7:

$$\frac{1 \cdot 2 + \dots}{\sqrt{1^2 + 4^2 + 1^2} \sqrt{2^2 + 3^2 + 0^2}}$$

Handwritten calculation for problem 8:

$$\frac{13}{18} - \frac{10}{4} = \frac{13}{18} - \frac{45}{18} = -\frac{32}{18} = -\frac{16}{9}$$



Handwritten calculation for problem 10:

$$-\left(\frac{4}{3}\right)^3 + 4\left(\frac{4}{3}\right)\left(\frac{4}{3}\right) - 2\left(\frac{4}{3}\right)^2 + 1$$

$$= -\frac{64}{27} + \frac{64}{9} - \frac{32}{9} + 1$$

$$= \frac{96}{32} - \frac{64}{32} + \frac{32}{32} + \frac{32}{32} = \frac{96 - 64 + 32 + 32}{32} = \frac{96}{32} = 3$$