大同大學 九十 學年度研究所招生入學考試試題

考試科目 ： 微積分　　所別 ： 事業經營研究所 第 1/1 頁

註 ： 本次考試不可以參考書籍及筆記；不可以使用字典；不可以使用計算器

(10 points for each problem)

1. If a function is defined as

$$f(x,y) = \begin{cases} 2(1-x), & 0 \leqq x \leqq 1,\ 0 \leqq y \leqq 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Find the function of $u = xy$.

2. Find the directional derivative of the function $f(x,y,z) = xyz$ at the point $\mathbf{p} = (-4,2,2)$ in the direction of the unit vector of $\mathbf{v} = (-1,4,7)$. Also find the direction and value of the maximum directional derivative at $\mathbf{p}$.

3. Find the shortest distance from the origin to the plane $x - y + z = 4$.

4. Sum the first n terms of the series whose k th term is $(3/4)^k(k+8)/[k(k+1)(k+2)]$.

5. Identify the conic of $x^2 + xy + y^2 = 2$ by the discriminant test and find the rotation angle so as to eliminate the xy-term.

6. Find the solution of the differential equation $y''' - 2y'' - 3y' = 3x^2 + \exp(-x)$.

7. Determine the linear dependence or independence of the following three vectors $\mathbf{u} = (1,1,1)$, $\mathbf{v} = (1,-1,3)$, and $\mathbf{w} = (1,0,1)$.
Express $\mathbf{z} = (4,3,2)$ as a linear combination of $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

8. Find the Taylor series of $\exp(x+y)$ about $(0,0)$.

9. Evaluate the integral over the region R: triangle bounded by $y = x$, $y = 2x$, and $x = 2$.

$$\int\int y / (x^2+y^2)\, dA.$$

10. Find the maximum value of $f(x,y) = \exp(xy)$ subject to the constraint $x^2 + y^2 = 8$.