大同大學 九十三 學年度研究所硕士班入學考試試題

10% for each problem

1. How to define a relation and a function between two sets X and Y? Find the domain of the function

$$y = (x^2 - x - 6)^{1/2}.$$

2. Use the ε-δ definition of a limit to prove that the limit of $(3x - 4)$ is 2 as $x \to 2$.

3. Find the limit of $y = (\tan x\ /\ x)^{1/x^2}$ as $x \to 0$.

4. Use the Trapezoidal Rule to approximate the value of the definite integral $f(x) = (x+1)^{-2}$ on $[1, 2]$ for $n = 4$ and compare the result with the exact value of the definite integral.

5. Find the general solution of $dy/dx - 2y/x - x = 0$.

6. The kth term of a series is $(k^2 - 1)^{-1}$ where $k \geq 2$. Find the sum of the first n terms of the series.

7. If $f(x) = (x^2 - 1)^{-1}$ find $f^{(990)}(0)$, the 990th derivative of $f(x)$ at $x = 0$.

8. The density function of the bivariate normal distribution is as follows:

$$f(x,y) = (k) \exp\{- [((x - \mu_x)/\sigma_x)^2 - 2\rho ((x - \mu_x)/\sigma_x)((y - \mu_y)/\sigma_y) + ((y - \mu_y)/\sigma_y)^2]/2(1 - \rho^2)\}$$

Where μ_x, μ_y, $\sigma_x > 0$, $\sigma_y > 0$, and ρ are parameters of the function and

$$k = [2\pi \sigma_x \sigma_y (1 - \rho^2)^{1/2}]^{-1}, \quad -\infty < x < \infty, \quad -\infty < y < \infty, \quad -1 \leq \rho \leq 1.$$

Find the contour curve(等高線)of the bivariate normal surface and determine the property of the curve. Is the curve a parabola, an ellipse, or a hyperbola? What happen if $\rho = 0$?

9. Find the projection of vector $\mathbf{p} = (1, 2, 2)$ onto vector $\mathbf{q} = (3, 4, 0)$ and the vector component of $\mathbf{p}$ orthogonal to $\mathbf{q}$.

10. Find the relative extrema of $f(x,y) = 2 + 2(2x - y)y - x^3$.