

# 大同大學 九十四 學年度研究所碩士班入學考試試題

考試科目：微積分 所別：事業經營研究所 共壹頁 第一頁

註：本次考試 不可以參考自己的書籍及筆記； 不可以使用字典； 不可以使用計算器。

1. If  $y(t)$  represents the size of a certain population at time  $t$  and it might be governed by the differential equation  $dy/dt = ky - (k/N)y^2$ , where  $k$  and  $N$  are constant and  $0 < y < N$ . Let  $y(0)$  be the initial population size. Find the general solution of the equation. (15%)
2. Find the integral of  $\exp(x) / (1 + \exp(2x))$ . (10%)
3. The  $f(x)$  is a probability density function of a continuous random variable  $X$  in the interval  $[a, b]$ , if  $f(x) \geq 0$  and the integral of this function is equal to 1 for all  $x$  in the interval. Find the constant  $k$  if the  $f(x)$  is a probability density function on  $(-\infty, +\infty)$ , where  $f(x) = k \cdot \exp(-x^2/2)$ . (12%)
4. Estimate the parameters  $a$  and  $b$  of the straight line  $f(x) = a + bx$  through the four points as showed below such that the sum of the squares of the differences of  $y$  and  $f(x)$ , i.e.,  $\sum(y - f(x))^2$ , for the four points is minimized. (15%)

x	1	2	3	5
y	6	6	7	9

5. Find all relative extrema for the function  $f(x, y) = y^4 - 2y^2 + x^2 - 17/16$ . (12%)
6. Find the volume of a right circular cone with height  $x_h$  and base radius  $r$ . (10%)
7. Find the area enclosed by  $f(x) = x^{1/2}$  and  $g(x) = x^3$  on  $[0, 2]$ . (12%)
8. Let  $R$  be the region bounded by the square with vertices  $(1, 1)$ ,  $(2, 0)$ ,  $(1, -1)$ ,  $(0, 0)$ . Evaluate the integral  $\int_R xy \, dx \, dy$ . (14%)