註：本次考試　不可以參考自己的書籍及筆記；　不可以使用字典；　可以使用計算器。

1. The magnitude of earthquakes recorded in a region can be modeled as having an exponential distribution with mean 3, as measured on the Richter scale. Of the next ten earthquakes to strike this region, what is the probability that at least one will exceed 5.0 on the Richter scale? (10 %)

2. The joint probability distribution of Y_1 and Y_2 is given by $p(y_1, y_2) = \binom{4}{y_1}\binom{3}{y_2}\binom{2}{3-y_1-y_2} \bigg/ \binom{9}{3}$, where y_1 and y_2 are integers, $0 \le y_1 \le 3$, $0 \le y_2 \le 3$, and $1 \le y_1 + y_2 \le 3$. Find $Cov(y_1, y_2)$. (10 %)

3. The joint probability density function of Y_1 and Y_2, which were the proportions of two components in a sample from a mixture of insecticide, is given below.

$$f(y_1, y_2) = \begin{cases} 2, & 0 \le y_1 \le 1 \\ 0, & elsewhere. \end{cases}, \quad 0 \le y_2 \le 1, \quad 0 \le y_1 + y_2 \le 1$$. Find $p(Y_1 \ge 1/2 \mid Y_2 \le 1/4)$. (10 %)

4. Let $Y_1, Y_2, ..., Y_9$ be a random sample of size 9 from a normal population with mean 0 and variance 1, and let $\overline{Y} = (\sum_{i=1}^{9} Y_i)/9$. Moreover, let Y_{10} be another independent observation from the same population. (20 %)

a. What is the distribution of $W = \sum_{i=1}^{9} Y_i^2$?

b. What is the distribution of $U = \sum_{i=1}^{9} (Y_i - \overline{Y})^2$?

c. What is the distribution of $3Y_{10}/\sqrt{W}$?

d. What is the distribution of $(Y_1 - Y_2)^2/2 + (Y_3 - Y_4)^2/2$?

5. What is a time series? Describe the components of a time series. (10%)

6. A standard regression package was run on the data including dependent variable y and two predictors, x1, x2, and the interaction term x1x2 with sample size n=30. The outputs are shown in the following tables.
   a. What is the population regression model and what is the sample regression model?
   b. What information or conclusion can you draw from each of the tables below? (20%)

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	0.788	0.621	0.577	5.45458

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	1267.403	3	422.468	14.199	0.000
	Residual	773.564	26	29.752		
	Total	2040.967	29			

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	4.940	7.525		.657	0.517
	x1	9.266	1.930	1.321	4.801	0.000
	x2	9.591	2.957	1.396	3.244	0.003
	x1x2	-1.947	0.857	-0.934	-2.273	0.032

Note:　Predictors: (Constant), x1x2, x1, x2, Dependent Variable: y.

〈背面繼續〉

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7. (1) Define the terms: factor, level, treatment, and simple main effect used in ANOVA. (2) How to examine your data before using the ANOVA? (3) Two factors A and B are investigated at a=2 (a1 and a2) and b=3 (b1, b2, and b3) levels, respectively. A standard ANOVA package was run on the data including the factors A, B and the dependent variable y with 5 observations for each of the six treatments (cells). What information or conclusion can you draw from the outputs shown in the following tables?　　(20%)

Descriptive Statistics

Dependent Variable: y

A	B	Mean	Std. Deviation	N
a1	b1	8.20	1.304	5
	b2	3.20	1.483	5
	b3	4.40	1.342	5
a2	b1	4.80	1.095	5
	b2	4.20	1.924	5
	b3	9.40	1.673	5

Levene's Test of Equality of Error Variances

Dependent Variable: y

F	df1	df2	Sig.
0.469	5	24	0.796

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.

Tests of Between-Subjects Effects

Dependent Variable: y

Source	S S	df	Mean Square	F	Sig.
A	5.633	1	5.633	2.522	0.125
B	60.800	2	30.400	13.612	0.000
A * B	88.267	2	44.133	19.761	0.000
Error	53.600	24	2.233		

Note:　R Squared = .743 (Adjusted R Squared = .689),　SS : Sum of Squares

Tests of Significance for simple main effects

Source	S S	df	Mean Square	F	Sig.
WITHIN CELLS	53.60	24	2.23		
A WITHIN b1	974.700	1	28.90	12.94	0.001
A WITHIN b2	2.50	1	2.50	1.12	0.301
A WITHIN b3	62.50	1	62.50	27.99	0.000

SS : Sum of Squares