考試科目：微積分　　　　　　所別：事業經營研究所　　　　第全頁

註：本次考試　不可以參考自己的書籍及筆記；　不可以使用字典；　可以使用計算器。

1. Find the limits:

   (1) $\lim\limits_{x \to 1} \dfrac{x-1}{x^2-1}$ (3%)

   (2) $\lim\limits_{t \to 1} \dfrac{t^2-3t-2}{t^2-1}$ (3%)

   (3) $\lim\limits_{x \to 1} \dfrac{\frac{1}{x+2}-\frac{1}{2}}{x}$ (4%)

2. Let $f(x) = \begin{cases} x^2-1, & x<3, \\ 2ax, & x \ge 3. \end{cases}$

   (1) What value should be assigned to a to make the function continuous at $x=3$? (5%)

   (2) Determine a complete graph of f for this value of a. (5%)

3. Find the absolute maximum and minimum values of
   $$f(x,y) = 2+2x+2y-x^2-y^2$$
   on the triangular plate in the first quadrant bounded by the lines $x=0$, $y=0$ and $y=9-x$. (10%)

4. If $f(x)$ is continuous on $[a, b]$, show that $2\int_a^b f(x)f'(x)dx = [f(b)]^2 - [f(a)]^2$. (10%)

5. Let $y=f(x)$, please find y such that $yy'+x^2 = 0$ and $f(0)=4$, where $y' = \dfrac{dy}{dx}$. (10%)

6. Determine the optimum quantity and the corresponding price for maximum profits for the Tatung Company.(10%)
   $\pi = TR - TC = (3.5q - 0.5q^2) - (0.04q^3 - 0.3q^2 + 2q + 1)$

7. Assume that a company manufactures a certain type of machine, and it estimates that each machine will generate a continuous income stream whose rate in the tth year of operation will be $15-2t$ million dollars per year. Assuming that the lifetime of a machine is about 7 years and that the money can be invested at an annual rate of 8%, compounded continuously, find the fair market price of each machine. (15%)

8. Find the equation of the plane through the points (3,0,0), (0,4,0) and (0,0,5). (10%)

9. A company manufactures two products, X and Y. The total cost in dollars for producing x units of product X and y units of product Y is given by the function
   $$C(x,y) = 0.25x^2 + 0.2y^2 + 6x + 7y + 800.$$
   (i) Find the cost of producing 20 units of X and 50 units of Y.
   (ii) Compute the partial marginal costs at (20, 50).
   (iii) Use linear approximation to estimate the cost of producing 22 units of X and 49 units of Y. Check the accuracy of the approximation by actually computing $C(22, 49)$ and comparing it to the estimated value. (15%)