

# 大同大學 98 學年度研究所碩士班入學考試試題

考試科目:微積分

所別:事業經營研究所

第全頁

註:本次考試 不可以參考自己的書籍及筆記; 不可以使用字典; 可以使用計算器。

- Tatung company estimates that when it spends  $x$  millions dollars to advertise its product, its annual revenue  $R$ , in millions of dollars, is modeled by the function  $R(x) = 500 - \frac{1000}{x+4}$ .
  - Compute  $\lim_{x \rightarrow 0} R(x)$  and  $\lim_{x \rightarrow \infty} R(x)$ . (4%)
  - If the company is currently spending \$30 million on advertising, would you recommend increasing it to \$40 million? And why? (6%)
- Suppose a small particle is moving along the  $x$ -axis in such a way that its position at time  $t$  is given by the formula  $x = t^{5/2} + 1$ .
  - What is its velocity when  $t=1$ ? When  $t=4$ ? (4%)
  - What is its average velocity between times 1 and 4? (6%)
- Find the global maximum and minimum, if any, of the function  $f(x) = xe^{-x^2/2}$  on the interval  $(-\infty, \infty)$ . (10%)
- Find the area of the region between the curves  $y = 9 - x^2$  and  $y = x^2 + 1$  from  $x=0$  to  $x=3$ . (10%)
- The function  $f(x, y, z) = 2x^2 + y^2 + z^2$  has a minimum subject to the constraint  $x + 3y - z = 7$ . Please use the method of Lagrange multipliers to find the minimum value. (10%)
- Universal Motors makes compact and midsized cars. The price function for compacts is  $p=17-2x$  (for  $0 \leq x \leq 8$ ), and the price function for midsized cars is  $q=20-y$  (for  $0 \leq y \leq 20$ ), both in thousands of dollars, where  $x$  and  $y$  are, respectively, the numbers of compact and midsized cars produced per hour. If the company's cost function is
$$C(x, y) = 15x + 16y - 2xy + 5$$
thousand dollars, find how many of each car should be produced and the prices that should be charged in order to maximize profit. Also find the maximum profit. (20%)
- A department store predicts that the number of bathing suits that it will sell during week  $t$  of the year (with  $t = 1$  meaning the first week of January) is given by the formula
$$S(t) = 400 - 400 \cos \frac{2\pi t}{52}.$$
Find the rate of change of the sales during the first week of May (week  $t = 18$ ). (10%)
- An aircraft company estimates its marginal revenue function for helicopters to be  $MR(x) = \sqrt{x^2 + 80x}(x + 40)$  thousand dollars, where  $x$  is the number of helicopters sold. Find the total revenue from the sale of the first 10 helicopters. (10%)
- Suppose that you have saved \$5000, and that you expect to save an additional \$3000 during each year. If you deposit these savings in a bank account paying 5% interest compounded continuously, find a formula for your bank balance after  $t$  years. (10%)