大同大學 89學年度研究所招生入學考試試題

考試科目 統計學　所別 資訊經營研究所　第1/3頁

註：本次考試 不可 參考自己的書籍及筆記 不可 查字典 可 使用計算器

Reference Data:

$z_{0.025} = 1.96$, $z_{0.05} = 1.645$.
$t_{0.025,18} = 2.101$, $t_{0.05,18} = 1.734$,
$t_{0.025,19} = 2.093$, $t_{0.05,19} = 1.729$,
$t_{0.025,20} = 2.086$, $t_{0.05,20} = 1.725$,
$t_{0.025,21} = 2.080$, $t_{0.05,21} = 1.721$,
$t_{0.025,22} = 2.074$, $t_{0.05,22} = 1.717$.
$\chi^2_{0.025,18} = 31.526$, $\chi^2_{0.05,18} = 28.869$, $\chi^2_{0.975,18} = 8.231$, $\chi^2_{0.95,18} = 9.390$,
$\chi^2_{0.025,19} = 32.852$, $\chi^2_{0.05,19} = 30.143$, $\chi^2_{0.975,19} = 8.906$, $\chi^2_{0.95,19} = 10.117$,
$\chi^2_{0.025,20} = 34.170$, $\chi^2_{0.05,20} = 31.410$, $\chi^2_{0.975,20} = 9.591$, $\chi^2_{0.95,20} = 10.851$,
$\chi^2_{0.025,21} = 35.478$, $\chi^2_{0.05,21} = 32.670$, $\chi^2_{0.975,21} = 10.283$, $\chi^2_{0.95,21} = 11.591$,
$\chi^2_{0.025,22} = 36.781$, $\chi^2_{0.05,22} = 33.924$, $\chi^2_{0.975,22} = 10.982$ $\chi^2_{0.95,22} = 12.338$.

Multiple Choice:

In the following, there are 20 problems. Each of them has only one correct choice and scores 5 points. Making a wrong choice on a problem, you will get −2 points for that problem. The minimum score of this test is zero.

Answer problem 1 and 2 using the following problem setting:
Let X be a random variable with pdf

$$f(x) = \begin{cases} k[1-(x-3)^2] & 2 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

1. The value of k =? (a) 0.25; (b) 0.5; (c) 0.75; (d) 1.
2. The value of $E[X]$ =? (a) 2.5; (b) 3; (c) 3.5; (d) 3.6.

Answer problem 3 to 5 using the following problem setting:
Two components of a minicomputer have the following joint pdf for their useful lifetimes X and Y:

$$f(x,y) = \begin{cases} xe^{-x(1+y)} & x \ge 0 \text{ and } y \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

3. What is the probability that the lifetime X of the first components exceeds 3? (a) e^{-3}; (b) $\frac{e^{-3}}{2}$; (c) $\frac{e^{-3}}{6}$; (d) $\frac{1}{3}$.
4. What is the probability that the lifetime Y of the second components exceeds 3? (a) $\frac{1}{2}$; (b) $\frac{1}{3}$; (c) $\frac{1}{4}$; (d) e^{-4}.
5. What is the probability that the lifetime of at least one component exceeds 3? (a) $\frac{1}{3}$; (b) $\frac{1}{4}+e^{-3}-\frac{1}{4}e^{-12}$; (c) $\frac{3}{4}-e^{-3}+\frac{1}{4}e^{-12}$; (d) $\frac{1}{4}+e^{-2}-\frac{1}{4}e^{-12}$.

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6. Let A and B are two independent events, and let A' and B' denote their complements, respectively. Which of the following statements is false: (a) $P(A|B) = P(A)$; (b) $P(A|B') = P(A)$; (c) $P(A \cup B) = P(A) + P(B)$; (d) $P(A \cap B') = P(A)(1 - P(B))$.

7. The probability of a man hitting a target is $\frac{1}{3}$. At least how many times must he fire so that the probability of hitting the target at least once is more than 90%: (a) 4; (b) 5; (c) 6; (d) 7.

8. Let $x_1, x_2, \ldots, x_{10}$ be a sample from a population. Suppose that $\sum_{i=1}^{10} x_i^2 = 297$ and sample variance s^2=36.9. Then, the sample mean $\bar{x}$ satisfies: (a) $48 < \bar{x} \le 50$; (b) $50 < \bar{x} \le 52$; (c) $52 < \bar{x} \le 54$; (d) $54 < \bar{x} \le 56$.

9. Let $X \sim N(\mu_1, \sigma^2)$, $Y \sim N(\mu_2, 2\sigma^2)$ be two independent and normally distributed random variables. Which of the following statements is false: (a) $Var[X+Y] = Var[X-Y]$; (b) $X - Y \sim N(\mu_1 - \mu_2, 3\sigma^2)$; (c) $Var[2X+Y] = 3Var[Y]$; (d) $Var[2X] = Var[Y]$.

10. z_α denotes the z critical value of an $N(0,1)$ population. Suppose that $0 < \alpha_1 < \alpha_2 < \alpha_3 < 0.5$ and $\alpha_2 - \alpha_1 = \alpha_3 - \alpha_2$. Then, (a) $z_{\alpha_1} - z_{\alpha_2} = z_{\alpha_2} - z_{\alpha_3}$; (b) $z_{\alpha_1} - z_{\alpha_2} > z_{\alpha_2} - z_{\alpha_3}$; (c) $z_{\alpha_1} - z_{\alpha_2} < z_{\alpha_2} - z_{\alpha_3}$; (d) all of the above statements are incorrect.

Answer problem 11 and 12 using the following problem setting:
Let a population have pdf $f(x) = \frac{1}{k^2}x, 0 \le x \le \sqrt{2}k$, and let $x_1, x_2, \ldots, x_n$ be a sample obtained from the population.

11. Let $\hat{k}_1$ be the MLE of parameter k. Then $\hat{k}_1$ =? (a) $\max(x_i)$; (b) $\frac{1}{2}\max(x_i)$; (c) $\sqrt{2}\max(x_i)$; (d) $\frac{\sqrt{2}}{2}\max(x_i)$.

12. Let $\hat{k}_2$ be the estimator of parameter k obtained using the method of moments. Then $\hat{k}_2$ =? (a) $\frac{3\sqrt{2}}{4}\bar{x}$; (b) $\frac{4\sqrt{2}}{3}\bar{x}$; (c) $\frac{\sqrt{3}}{2}\bar{x}$; (d) $\frac{2\sqrt{3}}{3}\bar{x}$.

Answer problem 13 to 15 using the following problem setting:
There are two populations $X \sim N(\mu_X, \sigma^2)$ and $X \sim N(\mu_Y, \sigma^2)$. The following table lists some sample statistics calculated from the random samples of the two populations:

	Sample Size	*Sample Mean*	*Sample Variance*
Population X	11	540	441
Population Y	11	554	225

13. The 95% confidence interval of σ^2: (a) (194.908, 694.401); (b) (197.108, 680.055); (c) (199.179, 667.092); (d) (331.04, 334.96);

14. The 95% confidence interval of $\mu_Y - \mu_X$ is: (a) (−2.138, 30.138); (b) (−2.185, 30.185); (c) (−2.231, 30.231); (d) (1.251, 26.749).

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15. Using the statistics to perform hypothesis test on $H_0 : \mu_X = \mu_Y$ versus $H_1 : \mu_X \neq \mu_Y$ at significance level of $\alpha = 0.05$. Then, the conclusion will be: (a) to reject H_0; (b) to reject H_1; (c) to reject both; (d) the statistics are inadequate.

Answer problem 16 and 17 using the following problem setting:
To perform a single-factor ANOVA, suppose that we get the following partially filled table:

Source of Variation	*df*	*Sum of Squares*	*Mean Square*	*f*
Treatments	? 2	0.0608	? 0.0304	? 0.561
Error	12	? 0.3696	0.0308	
Total	14	? 0.4304		

16. Let m denote the number of populations (or treatments) involved in the ANOVA test, and let n denote the sum of sample sizes obtained from the m populations. Then, $m + n =$ (a) 14; (b) 15; (c) 16; (d) 18.

17. At significance level of $\alpha = 0.05$, we can conclude: (a) $\mu_1 = \mu_2 = \cdots = \mu_m$; (b) there are some i, j, where $1 \le i < j \le m$, such that $\mu_i \neq \mu_j$; (c) $\sigma_1^2 = \sigma_2^2 = \cdots = \sigma_m^2$; (d) since the value of critical value $f_{0.05,\nu_1,\nu_2}$ is not available, no conclusion can be drawn.

Answer problem 18 to 20 using the following problem setting:
Given $n = 15$ data points (x_i, y_i)'s, i=1, 2, ..., 15, the following quantifies were computed:

$$S_{xx} = 50, S_{yy} = 25, S_{xy} = -30, \bar{x} = 1.3, \bar{y} = 27$$

18. To fitting the data points to a least squares line $y = \beta_1 + \beta_2 x + \epsilon$, $\hat{\beta}_1 =$ (a) $\frac{1}{2}$; (b) $-\frac{3}{5}$; (c) $-\frac{6}{5}$; (d) $\frac{270}{13}$.

19. The coefficient of determinations $r^2 =$? (a) 0.72; (b) 0.50; (c) 0.45; (d) 0.36.

20. In linear regression, we assume $\epsilon \sim N(0, \sigma^2)$. Using the above statistics, we get $\hat{\sigma} =$? (a) 1.52; (b) 1.62; (c) 1.72; (d) 1.82.