大同大學 九十二 學年度研究所碩士班入學考試試題

考試科目：資料結構與演算法　　所別：資訊經營研究所　　

註：本次考試 不可以參考自己的書籍及筆記； 不可以使用字典； 不可以使用計算器。

1. (12 pts) Write a recursive routine to compute Fibonacci numbers (0, 1, 1, 2, 3, 5, 8, 13, 21, 34, …) and show that its running time grows exponentially. Explain, however, why stack space is not exponential if your routine is run for $N = 100$.

2. (12 pts) For the following program fragment, give an analysis of the running time (in Big-Oh notation):

```
sum = 0;
for( i = 1; i < n; i++)
    for( j = 1; j < i * i; j++ )
        if( j % i == 0 )
            for( k =0; k < j; k++ )
                sum++;
```

3. (12 pts) Given the alphabet and frequency table below, construct the Huffman tree and fill in the code of each symbol in the alphabet, and see if you can decode 1110100010111011.

Symbol	Frequency	Code	Symbol	Frequency	Code	Symbol	Frequency	Code
A	14		*D*	12		*G*	6	
B	5		*E*	25		*H*	1	
C	7		*F*	3		*I*	15	

4. (12 pts) For the original file of integers: 37 92 33 57 25 12 86 48, create a heap of size 8 (stored in an array) and show the heapsort procedure to adjust the heap tree until sorted.

5. (14 pts) The following keys/values are to be inserted in a hash table.
   25, 42, 96, 101, 102, 162, 196, 200
   Use the division method of hashing with a table size of 11. For resolving hash clashes, use double hashing. Show the final table and the total number of probes in building the table (indicate the number of probes during each insertion).

   Hint: • $rh(i, key) = (i + h2(key)) \% tablesize$
   • $h2(key) = \begin{cases} q \% tablesize, & \text{where } q * tablesize + key \% tablesize = key \\ 1, & \text{if } q \% tablesize = 0 \end{cases}$

6. (14 pts) (a) How many comparisons are required to sort 6 integers in the *worst* case and in the *average* case using any comparison-based algorithm? (b) Give algorithms to sort 4 elements in 5 comparisons.

7. (12 pts) Determine the running time of the quicksort implementation, e.g., "median-of-three with cutoff" algorithm, for a) sorted input, b) reverse input, c) random input. How about when the pivot is chosen as the first element for a), b) and c)?

8. (12 pts) Solve the following recurrence (answers in Big-Oh notation):

$$T(n) = \begin{cases} c, & \text{if } n < 4 \\ c + 2T\left(\left\lfloor n/4 \right\rfloor\right), & \text{if } n \geq 4 \end{cases}$$