Multiple Choices Questions (1-5): [8 points each]

1. Solve $2a_n - 3a_{n-1} = 0$,　　　$n \geq 1$,　　　$a_4 = 81$. $a_n = C\left(\dfrac{3}{2}\right)^n, C = ?$

   (a) 2　(b) 4　(c) 6　(d) 8　(e) none of the above.

2. $S = \{(x, y) \mid 3x + 2y = 100, x, y \in N\}$ ； the discrete probability, P, to pick an element (x, y) in S such that $x > y$ is
   (a) $P > 0.2$　(b) $P > 0.3$　(c) $P > 0.4$　(d) $P > 0.5$　(e) $P > 0.6$

3. Find the number of integer solutions to $x_1 + x_2 + x_3 = 20; x_1 \geq 3; x_2 \geq 2; x_3 \geq 0$.
   (a) 130　(b) 136　(c) 126　(d) 120　(e) none of the above.

4.-5. Use the following to answer questions 4-5:
   Use the definition of big-oh to prove **$n!$ is $O(n^n)$**. Determine the values of the "witness" C and k.
4. (a) C=0　(b) C=1　(c) C=2　(d) C=n　(e) none of the above.
5. (a) k=0　(b) k=1　(c) k=2　(d) k=n　(e) none of the above.

Proof and Calculations　(6-10:)

6. (a) What is the power set of the set {0, 1, 3}? (5 points)
   (b) Determine the set of all orderd pairs of the Cartesian product of A = {0, 1} and B = {1, 3}? (5 points)

7. Show that the set of odd positive interger is a countable set.　(10 points)

8. (a) Find a recurrence relation, {a_n}, and give initial conditions for the number of bit strings of length n that do not have two consecutive 0s.　(10 points)
   (b) Show the conditions such that {a_n} satisfies the same recurrence relation as the Fibonacci sequence.　(10 points)

9. For $n \in Z^+$, find in $(1 + x + x^2)(1 + x)^n$ the coefficient of x^r for $0 \leq r \leq n + 2, r \in Z$.　(10 points)

10. For $n \geq 0$, let a_n count the number of ways a sequence of 1's and 2's will sum to n. For example, $a_3 = 3$ because 1+1+1=3, 1+2=3, and 2+1=3. Find and solve a recurrence relation for a_n.　(10 points)