

3k-5k-2

大同大學 九十 學年度 轉學考試 試題

考試科目：離散數學 系別：資訊工程學系 級別：三年級 第 / 頁，共 / 頁

註：本次考試不可以參考自己的書籍及筆記； 不可以使用字典； 不可以使用計算器。

- Provide a combinatorial argument to show that if n and k are positive integers with $n = 3k$, then $n!/(3!)^k$ is an integer. (5%)
 - Generalize the result of part(a). (5%)
- Let $p(x, y)$ denote the open statement " x divides y ," where the universe for each of the variables x, y comprises all integers. Determine the truth value of each of the following statements; if a quantified statement is false, provide an explanation or a counterexample. (12%)

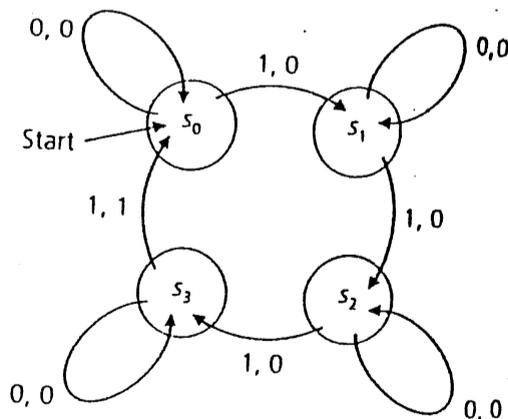
 - $\forall y \exists x p(x, y)$
 - $\exists y \forall x p(x, y)$
 - $\forall x \forall y [(p(x, y) \wedge p(y, x)) \rightarrow (x = y)]$

$n! = (n \times (n-1) \times (n-2) \times \dots \times 1)$
 $= 3k \times (3k-1) \times (3k-2) \times \dots$
- A professor has two dozen introductory textbooks on computer science and is concerned about their coverage of the topics (A) compilers, (B) data structures, and (C) operating systems. The following data are the numbers of books that contain material on these topics.
 $|A| = 8, |B| = 13, |C| = 13, |A \cap B| = 5$
 $|A \cap C| = 3, |B \cap C| = 6, |A \cap B \cap C| = 2$

 - How many of the textbooks include material on exactly one of these topics? (4%)
 - How many do not deal with any of the topics? (4%)
 - How many have no material on compilers? (4%)
- For $n \geq 0$, let F_n denote the n th Fibonacci number. Prove that
 $F_0 + F_1 + F_2 + \dots + F_n = \sum_{i=0}^n F_i = F_{n+2} - 1$. (10%)
- Determine the best "big-Oh" form of the following functions $f: \mathbf{Z}^+ \rightarrow \mathbf{R}$. (8%)

 - $f(n) = 5n^2 + 3n \log_2 n$
 - $f(n) = 2 + 4 + 6 + \dots + 2n$
- For $\Sigma = \{0, 1\}$ determine whether the string 00010 is in each of the following languages (taken from Σ^*). (6%)
(a) $\{0, 1\}^*$; (b) $\{0, 0\}^* \{1, 0\}^*$; (c) $\{00\} \{0\}^* \{1, 0\}$
- Let M be the finite state machine in the following figure.

 - Find the state table for this machine. (3%)
 - Explain what this machine does. (3%)
 - How many distinct input strings x are there such that $\|x\| = 8$ and $v(s_0, x) = s_0$? How many are there with $\|x\| = 12$? (6%)



F_0
 F_1
 $F_1 + F_0$
 $F_1 + F_1 + F_0$
 $F_1 + F_1 +$

- If $A = \{w, x, y, z\}$, determine the number of relations on A that are (a) reflexive; (b) symmetric; (c) antisymmetric; (d) reflexive and contain (x, y) . (12%)
- Find the generating function for the number of integer solutions to the equation $c_1 + c_2 + c_3 + c_4 = 20$ where $-3 \leq c_1, -3 \leq c_2, -5 \leq c_3 \leq 5$, and $0 \leq c_4$. (10%)
- What is the Hamilton cycle and what is the Hamilton path in a graph $G=(V,E)$? (8%)