大同大學 九十一 學年度 轉學考試 試題

考試科目：離散數學　系別：資訊工程學系　級別：三年級 第 / 頁，共 / 頁

註：本次考試不可以參考自己的書籍及筆記； 不可以使用字典； 不可以使用計算器。

1. (8%) In the complete expansion of $(a+b+c+d)(e+f+g+h)(u+v+w+x+y+z)$ one obtains the sum of terms such as agw, cfx, and dgv. How many such terms appear in this complete expansion?

2. (10%) Determine the number of integer solutions for $x_1 + x_2 + x_3 + x_4 + x_5 < 40$, where $x_i \geq -3$, $1 \leq i \leq 5$.

3. (8%) For primitive statements p, q, is $(p \vee q) \rightarrow [q \rightarrow (p \wedge q)]$ a tautology?

4. (10%) Validate the argument $[(p \rightarrow q) \wedge (\neg r \vee s) \wedge (p \vee r)] \rightarrow (\neg q \rightarrow s)$. You must give the reasons for each step.

5. (8%) If $A = \{a, b, d\}, B = \{d, x, y\}, C = \{x, z\}$, How many proper subsets are there for the set $(A \cap B) \cup C$? How many for the set $A \cap (B \cup C)$?

6. (10%) Prove that for all $n \in Z^+, n > 3 \Longrightarrow n^2 < 2^n$.

7. (9%) Let $A = \{10, 11\}, B = \{00, 1\}$ be languages for the alphabet $\Sigma = \{0, 1\}$. Determine each of the following:
(a) AB; (b) BA; (c) B^2.

8. (5%) Given 8 Pascal books, 17 FORTRAN books, 6 Java books, 12 COBOL books, and 20 C++ books, how many of these books must we select to insure that we have 9 books dealing with the same computer language?

9. (10%) In the following program segments, we define the time-complexity function $f(n)$ to be the number of times the statement $sum := sum + 1$ is executed. Determine the best "big-Oh" form for f.

(a)
```
begin
  sum:= 0
  for i:= 1 to n do
    for j := 1 to n*n do
      sum := sum + 1
end
```

(b)
```
begin
  sum:= 0
  i:= n
  while i > 0 do
    begin
      sum := sum + 1
      i = int(i/2)
    end
end
```

10. (12%) If $A = \{w, x, y, z\}$, determine the number of relations on A that are
(a) reflexive; (b) symmetric; (c)antisymmetric; (d)reflexive and contain (x, y).

11. (10%) In a certain area of the countryside are four villages. An engineer is to devise a system of two-way roads so that after the system is completed, no village will be isolated. In how many ways can he do this?