1. (10%) Find the general solution to the equation

$$6x - ye^{xy} + (e^y - xe^{xy})y' = 0 \; .$$

2. (10%) Solve the initial value problem to the equation

$$y' + \frac{2}{x+1} y = 3; \; y(0) = 5.$$

3. (15%) Find the general solution to the equation

$$y'' - 2y' + y = 3x + 25\sin(3x) \; .$$

4. (15%) Find the eigenvalues and corresponding eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & -2 & 0 \\ 0 & 0 & 0 \\ -5 & 0 & 7 \end{pmatrix}.$$

5. (15%) Let $f(x) = 5\sin(3x) - 3e^{|x|}$ for $-\pi \le x \le \pi$. Write the Fourier series of $f(x)$ on $[-\pi, \pi]$.

6. (15%) Compute the inverse Laplace transform of $\dfrac{4}{s(s+2)^2}$.

7. (20%) Solve the boundary value problem

$$\frac{\partial^2 y}{\partial t^2} = 4\frac{\partial^2 y}{\partial x^2} \; \text{ for } \; 0 < x < 2, \, t > 0;$$

$$y(0,t) = y(L,t) = 0 \; \text{ for } \; t > 0,$$

$$y(x,0) = \sin(6\pi x), \; \frac{\partial y}{\partial t}(x,0) = 5x \; \text{ for } \; 0 < x < 2.$$